Math Foundations: The Magic of Math

2018 - 1 hour presentation

Equals Mathematics

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Dr. Cain, Dr. Faulkner, and NCDPI

PISA (2012): Context

Program for International Student Assessment

USA ranked third in the OECD sample in per capita GDP

USA ranked fourth in the OECD sample in per student spending

The share of students from disadvantaged backgrounds is within the average range of the OECD sample

OECD, 2014

PISA (2012): Results

Program for International Student Assessment

The USA average score was 27th out of 34 countries*

26% of USA students scored below the baseline level of proficiency

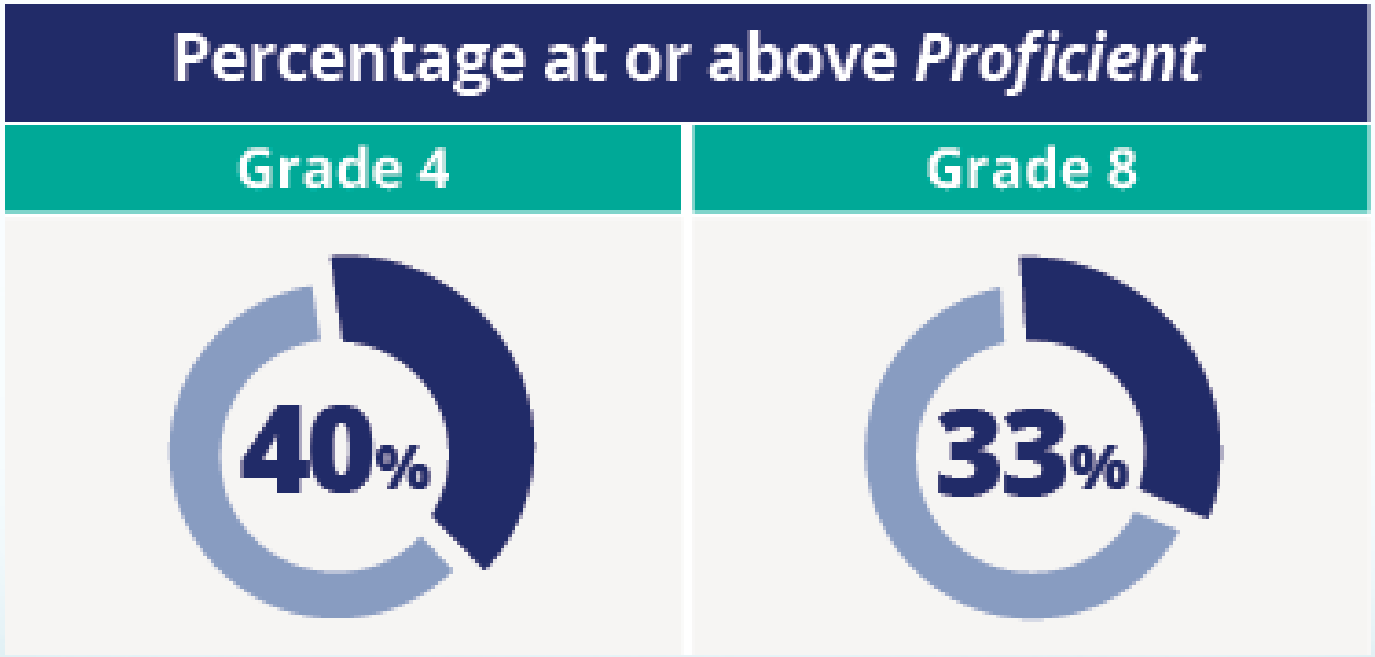
9% of USA students scored within the top proficiency level

*of OECD participating countries

OECD, 2014

NAEP: 2015

National Assessment of Educational Progress



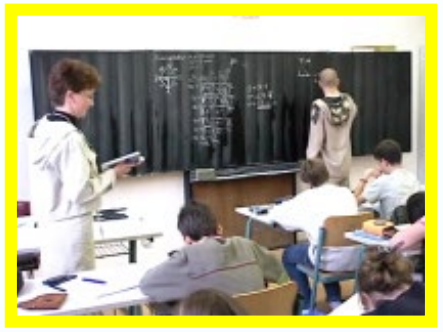
National Center for Educational Statistics, 2015

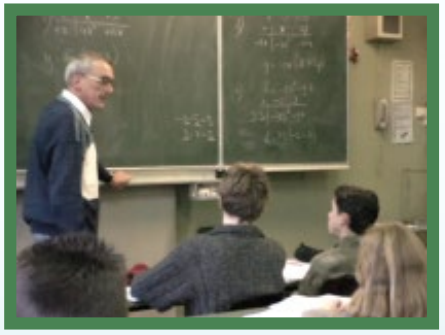
TIMSS

Trends in International Mathematics and Science Study

US Teachers		Hong Kong, Singapore, Japan
Learning terms and practicing procedures	Instructional focus	Structured problem solving
Covers 80% of tested topics	Pace	About half the tested topics
Mile wide, inch deep	Curriculum	Greater depth and coherence
How can I teach my students to get the answer to this problem?	Teachers plan by asking…	How can I use this exercise to teach mathematics they don't already know?

International Research


TIMSS Video Studies

- 1995 Video Study
  - Japan, Germany, US
  - Teaching Style Implicated
- 1999 Video Study
  - US, Japan, Netherlands, Hong Kong, Australia, Czech Rep.
  - Implementation Implicated

Stigler & Hiebert, 2004

Work Space

High Achieving Countries

MAKE CONNECTIONS

United States

TEACHES PROCEDURES

Structures and Connections

What is 4^2 ?

Procedure versus Structure/Connections

Make a square out of your 4 unit linear side

Work Space

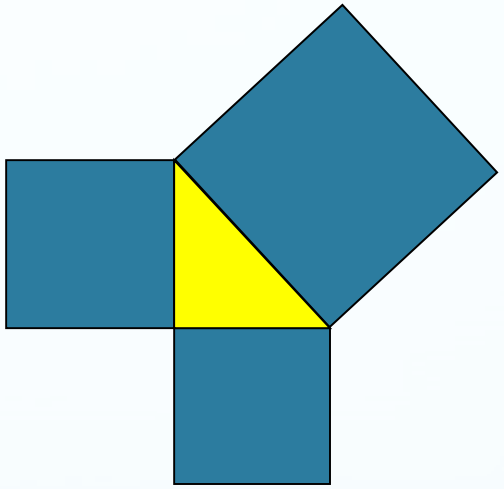
Exponents and CONNECTIONS

Square Roots!

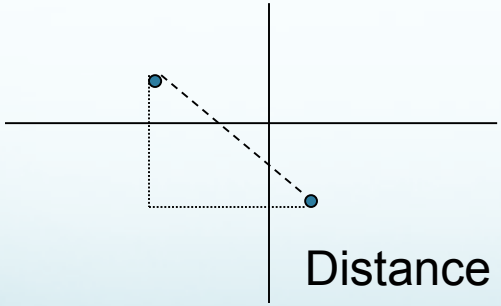
$\sqrt{16} = 4$

The length of one side!

Geometry and Measurement



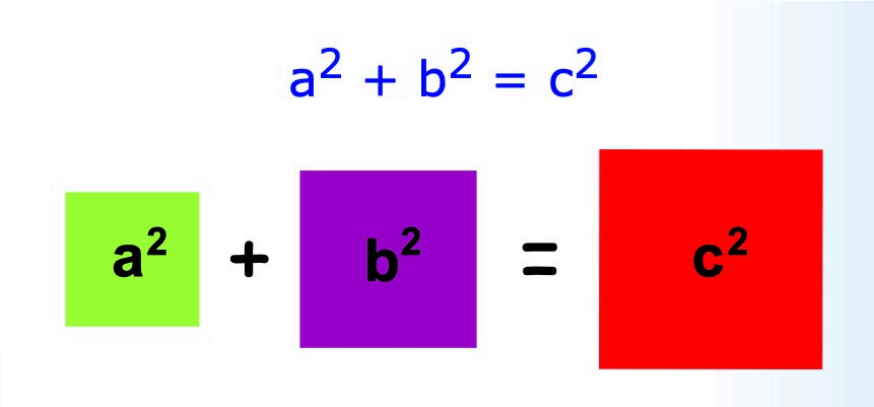
Pythagorean
Special Triangles
Trigonometry



Distance Formula

Connections

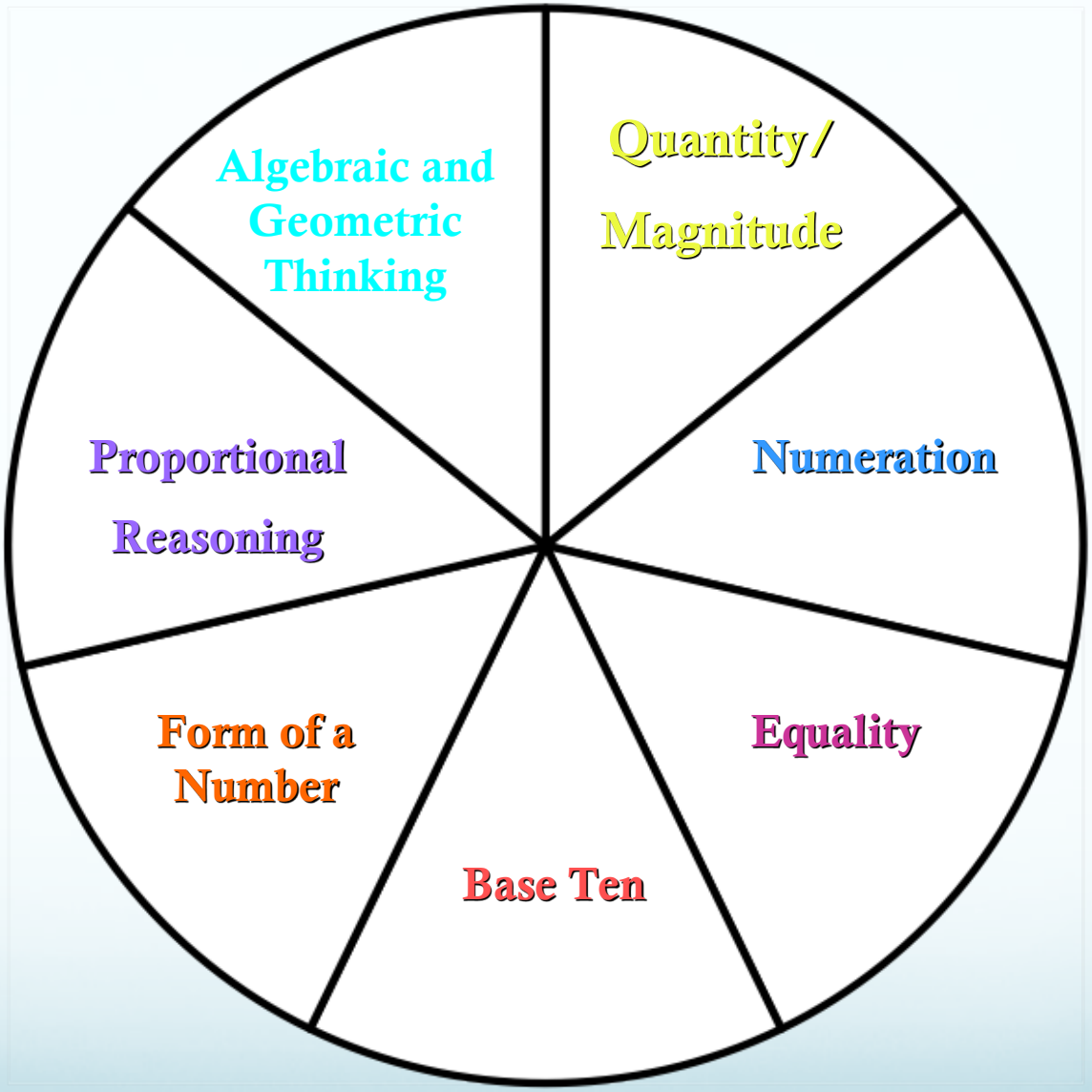
4^2



$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Components of Number Sense Overview

What’s the “Big Idea”?



Components of Number Sense (almost!)

Understanding and Instruction

We can only instruct our students as well as we understand the mathematics:

The better we understand the math, the better decisions we will make regarding what the student needs to achieve and how to instruct the student!

Knowing and Teaching Elementary Mathematics (Liping Ma)

- Compared and contrasted the pedagogy of Chinese and American teachers
- Found that American teachers were much weaker in content and conceptual knowledge
- Found American teachers teach procedurally rather than being driven by the logic of the mathematics (implementation)
- Ma presented information through teacher responses to elementary math questions

Defining Issue in Implementation

…is the teacher's own understanding of Mathematics.

Liping Ma (1999)

Problem #1 Subtraction

$$\begin{array}{r} 72 \\ -15 \\ \hline \end{array}$$

How would you approach this type of problem if you were teaching second grade?

Problem #1
Subtraction with Regrouping

American Teachers — Procedural approach

- The “pedagogic insight "of the teachers
  - Once the student can take a ten from the tens place and turn it into 10 ones, then they can address the problem correctly. Problem solved.
  - Manipulatives suggested to explain this step only.
  - Manipulatives sometimes used in a way that they did not actually demonstrate process of regrouping.

Problem #1
Subtraction with Regrouping

Chinese Teachers — Decomposing and Composing a Higher Value Unit

- Pedagogical insight: implemented the package of critical information embedded in subtraction
- Saw this problem as connected to addition through composing and decomposing units
- Demonstrated multiple ways of regrouping
- Found the opportunity to explore the basics of our base ten system

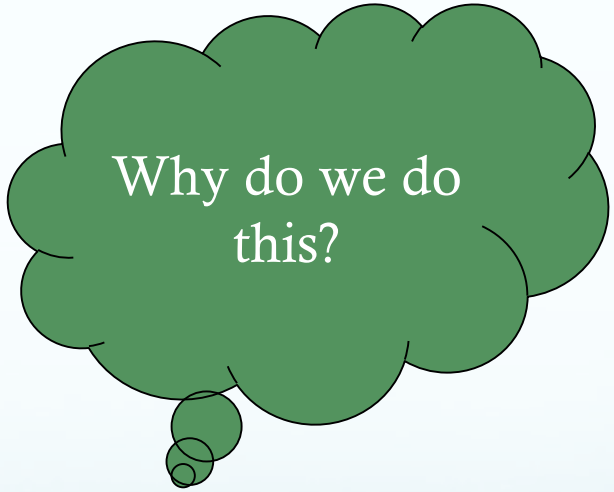
$$\begin{array}{r} 15 \\ \underline{-\ 8} \\ 7 \end{array}$$

15 - 8

15

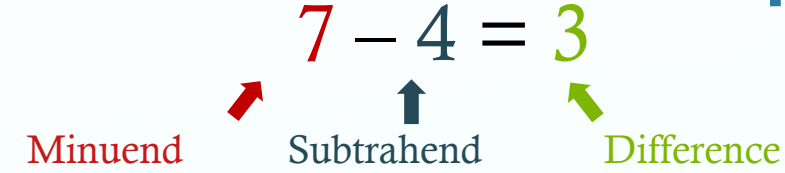
~~1~~5
- 8

7



Cain, Faulkner, & Fanelli

Subtraction: Decomposition



Minuend

- Is the first number in the subtraction problem.

Subtrahend

- Is the second number or the number that is being subtracted in the subtraction problem.

Difference

- Is the final answer after the subtrahend has been subtracted from the minuend.

Cain, Faulkner, & Fanelli

Approaching Computation

Achievement Level	Counting All	Counting On	Known Facts	Derived Facts
High Achieving	0%	9%	30%	61%
Low Achieving	22%	72%	6%	0%

Gray & Tall, 1994

Expanded Form

Using Expanded Notation

$$18 = 10 + 8$$

Subtraction: Decomposition

Using Expanded Notation

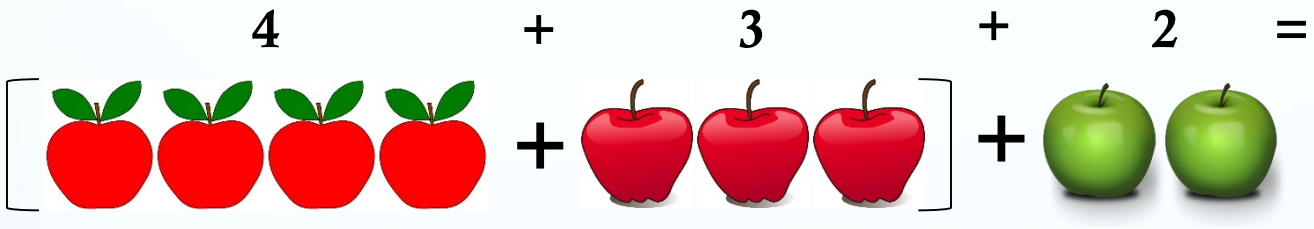
$$18 - 9$$

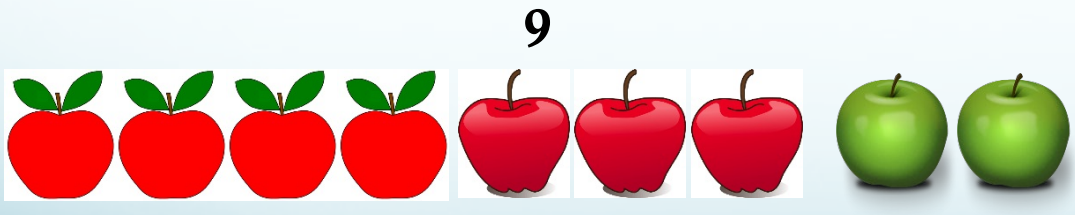
$$\begin{array}{r} 10 + 8 \\ -\ (9 + 0) \\ \hline \end{array}$$

$$1 + 8 = 9$$

Subtraction: Decomposition

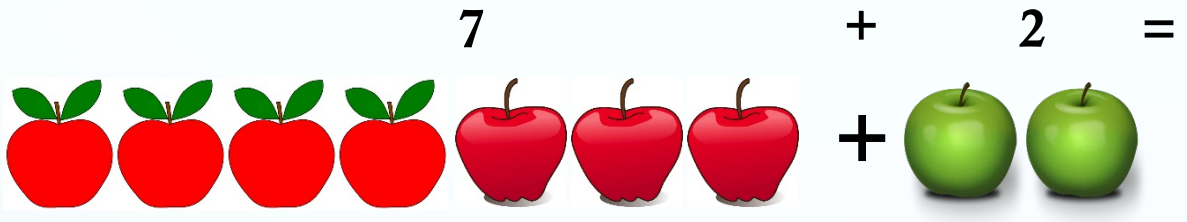
(4 + 3) + 2

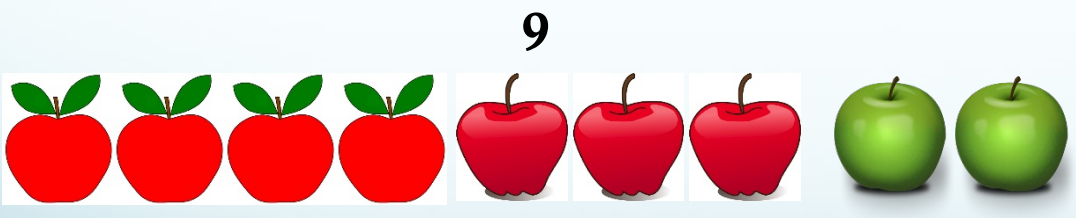




Subtraction: Decomposition

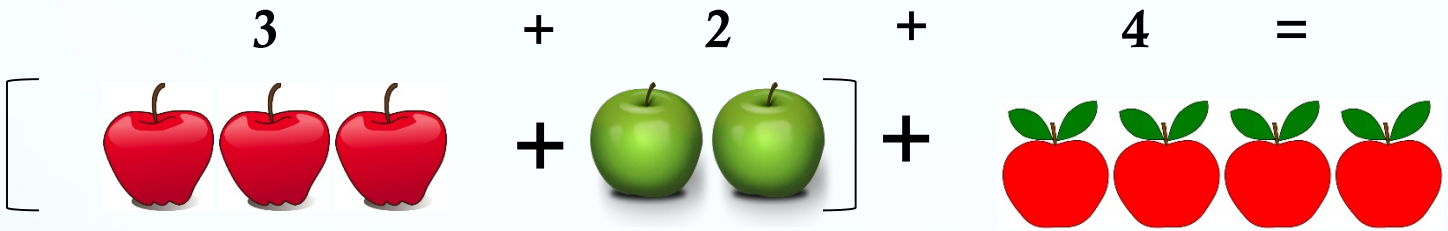
7 + 2

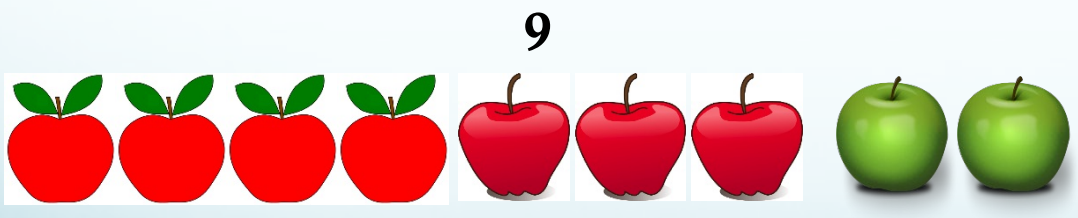




Subtraction: Decomposition

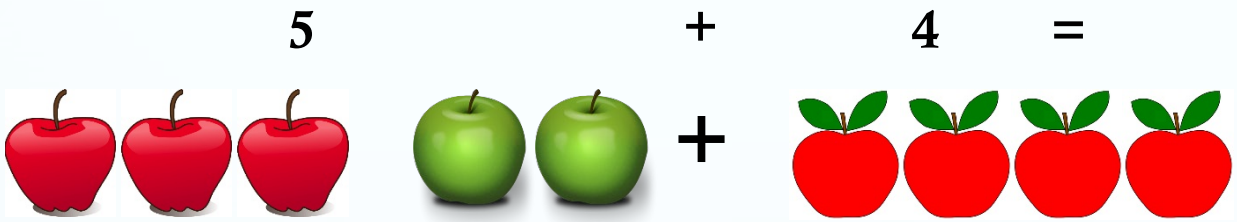
(3 + 2) + 4

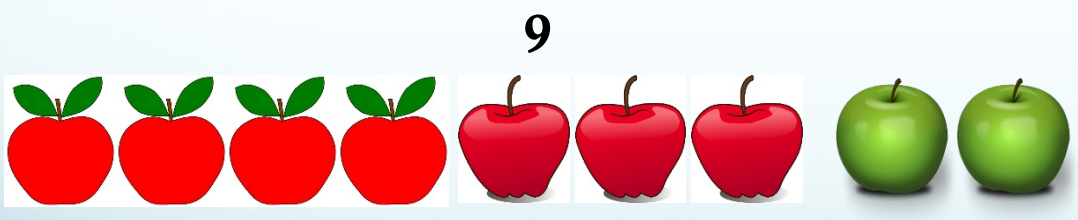




Subtraction: Decomposition

5 + 4





Subtraction: Decomposition

18 - 9

Using Decomposition instead of, or in addition to, Expanded Notation

$$(10 + 5 + 3)$$
$$-\,(\ 9 + 0 + 0)$$

$$1 + 5 + 3 = 9$$

Cain, Faulkner, & Fanelli

Subtraction: Decomposition

$$18 - 9$$

$$\begin{aligned} &(5 + 5 + 5 + 3) \\ -\,&(5 + 4 + 0 + 0) \\ \hline \\ &0 + 1 + 5 + 3 = 9 \end{aligned}$$

Subtraction: Decomposition

$$25 - 7$$

$$(20 + 5)$$
$$-\,(\ \ 7 + 0)$$

Expanded Notation doesn't get a student as far here.

Try other ways to decompose the number.

Cain, Faulkner, & Fanelli

Subtraction: Decomposition

$$25 - 7$$

$$\begin{array}{r} (10 + 10 + 5) \\ -\,(\ 7 + \ \ 0 + 0) \\ \hline \end{array}$$

$$3 + 10 + 5 \quad = \quad 18$$

Subtraction: Decomposition

$$25 - 7$$

$$\begin{array}{r} (10 + 10 + 5) \\ -\,(\ 5 + \ \ 2 + 0) \\ \hline \end{array}$$

$$5 + 8 + 5 = 18$$

Subtraction: Decomposition

$$127 - 25$$

$$\begin{array}{r} (100 + 25 + 2) \\ -\,(\quad 0 + 25 + 0) \\ \hline \end{array}$$

$$100 + 0 + 2 = 102$$

Work Space

Subtraction: Keeping the Distance

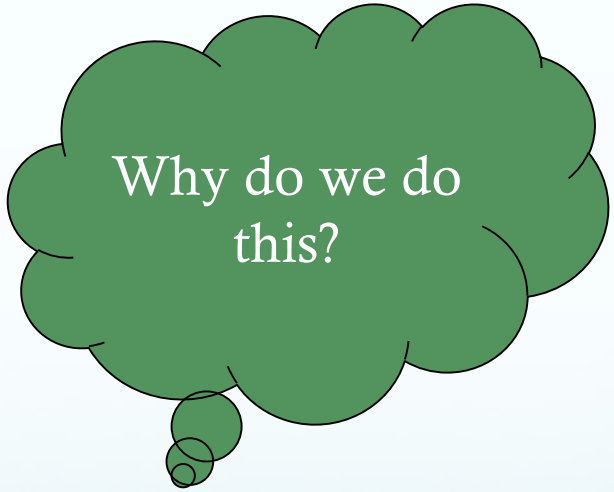
12 - 9

12

~~1~~2

- 9

3



Cain, Faulkner, & Fanelli

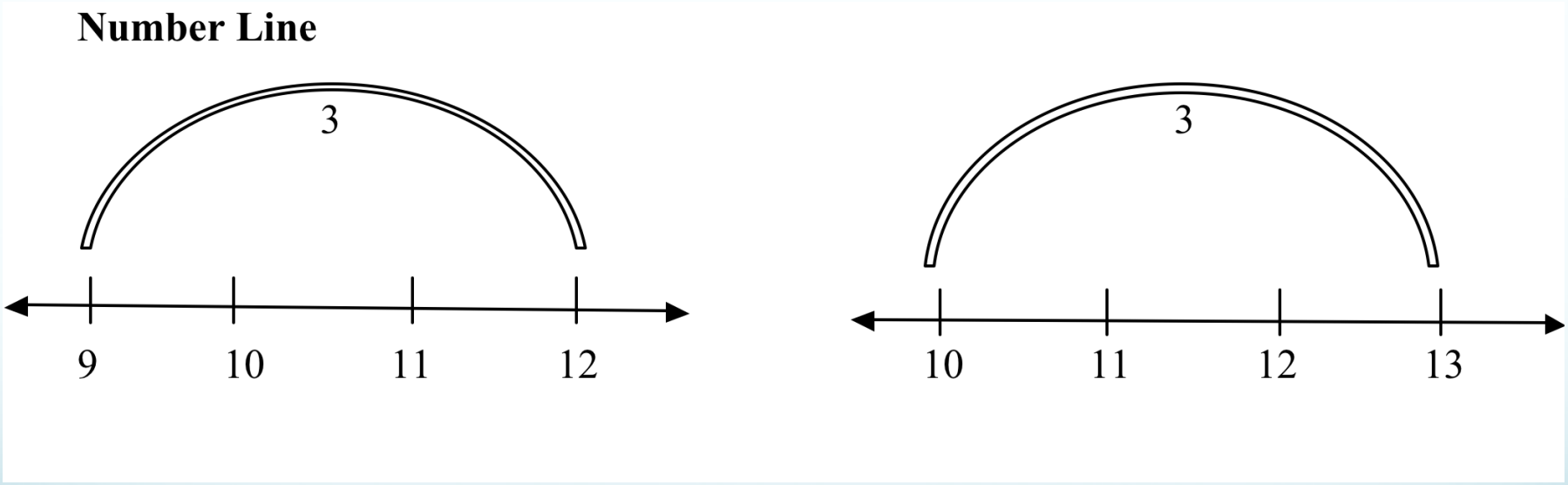
Subtraction: Keeping the Distance

12 - 9

Move each number on the number line in order to make the computation easier.

Subtraction: Keeping the Distance

12 - 9



Subtraction: Keeping the Distance

15 - 7

15

~~1~~5

- 7

8

Subtraction: Keeping the Distance

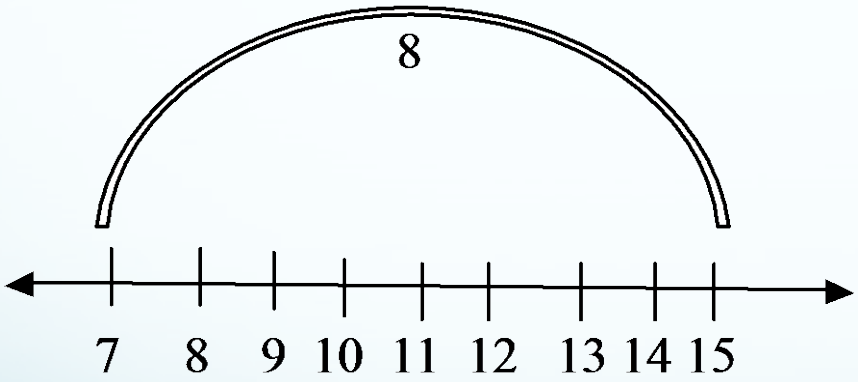
15 - 7

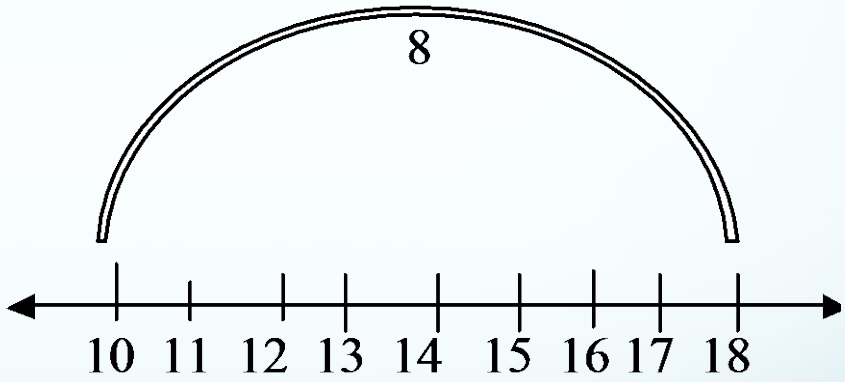
Move each number on the number line in order to make the computation easier.

Subtraction: Keeping the Distance

15 - 7

Number Line





Work Space

Subtraction: The Number Between

1. Choose a number that is in between the minuend and subtrahend.
2. Look to see how far the minuend is from that number.
3. Look to see how far the subtrahend is from that number.
4. Add those distances together to find the difference.

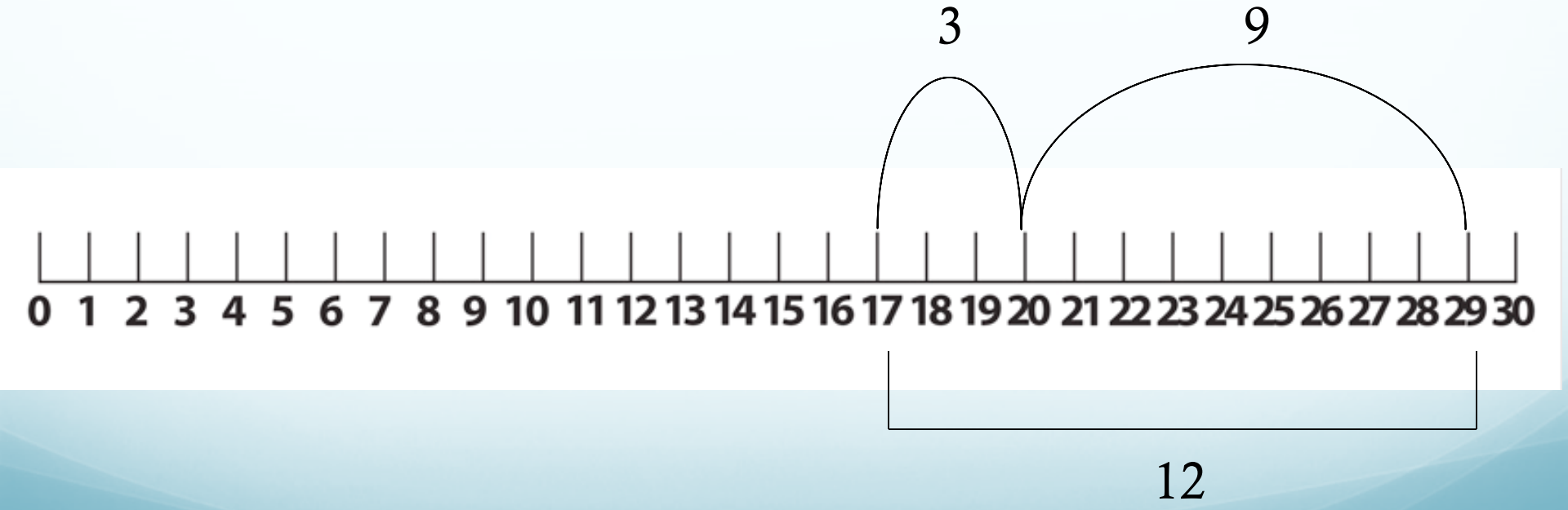
Subtraction: The Number Between

29 - 17

Original	Chosen #	Distance from original to chosen number
29 ←	20	9
- 17 →	20	+ 3
		12

Subtraction: The Number Between

29 - 17



Subtraction: The Number Between

170 - 145

Original	Chosen #	Distance from original to chosen number
170 ⟵	150	20
-145 ⟶	150	+ 5
		25

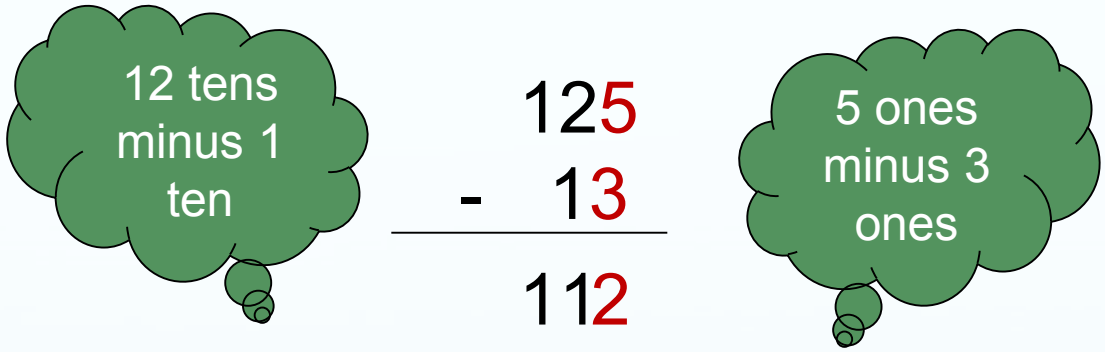
Work Space

Subtraction: Using the Correct Base 10 Language

Subtraction: Using the Correct Base 10 Language

$$\begin{array}{r} 125 \\ -\ \ 13 \\ \hline \end{array}$$

Subtraction: Using the Correct Base 10 Language



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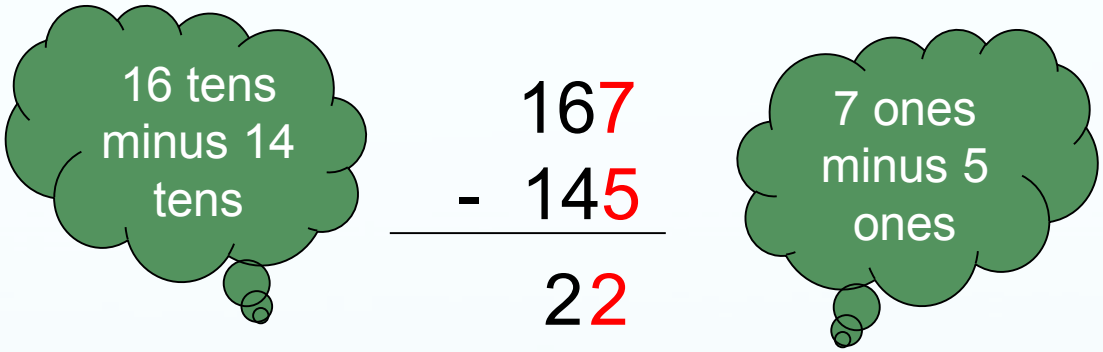
Subtraction: Using the Correct Base 10 Language



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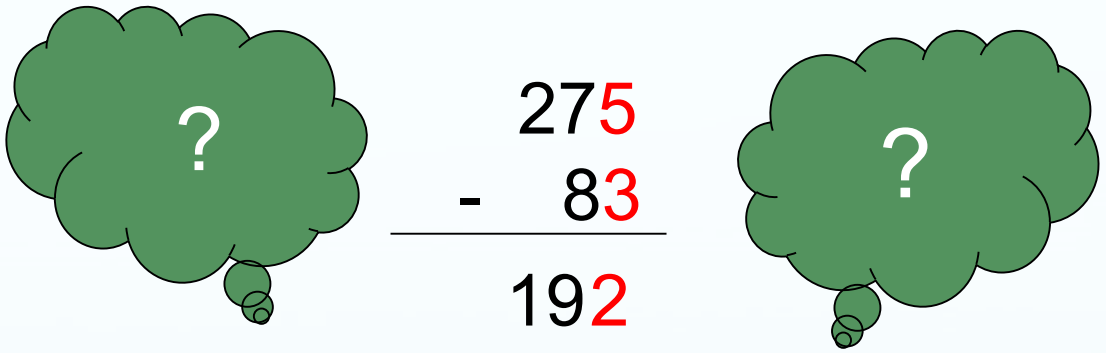
Subtraction: Using the Correct Base 10 Language



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Subtraction: Negatives

$$\begin{array}{r} 17 \\ -\ 9 \\ \hline \end{array}$$

Subtraction: Negatives

$$\begin{array}{r} 17 \\ -\ 9 \\ \hline -2 \end{array}$$

Subtraction: Negatives

$$
\begin{array}{r}
17 \\
-\ 9 \\
\hline
10 + (-2)
\end{array}
$$

Subtraction: Negatives

$$\begin{array}{r} 17 \\ -\ 9 \\ \hline \end{array}$$

$$10 + (-2) = 8$$

$$\begin{array}{r} 17 \\ -\ 9 \\ \hline 8 \end{array}$$

You Try

$$\begin{array}{r} 128 \\ -\ 94 \\ \hline \end{array} \qquad \begin{array}{r} 27 \\ -\ 18 \\ \hline \end{array}$$

Math “Facts” vs. Subtraction within 20

- U.S. deals with problems like 12-5, 15-7, etc., as FACTS to be memorized.
- It does help to have these memorized, however, PEDAGOGICALLY, there’s more to it.
- Chinese teach these “facts within 20” as the entry point for understanding our number system (develop “number sense”, emphasize base ten system)

Develop an Understanding of Base 10 and Equal Exchange

16-4	14-7
12-8	17-3
13-5	19-9
18-9	15-6

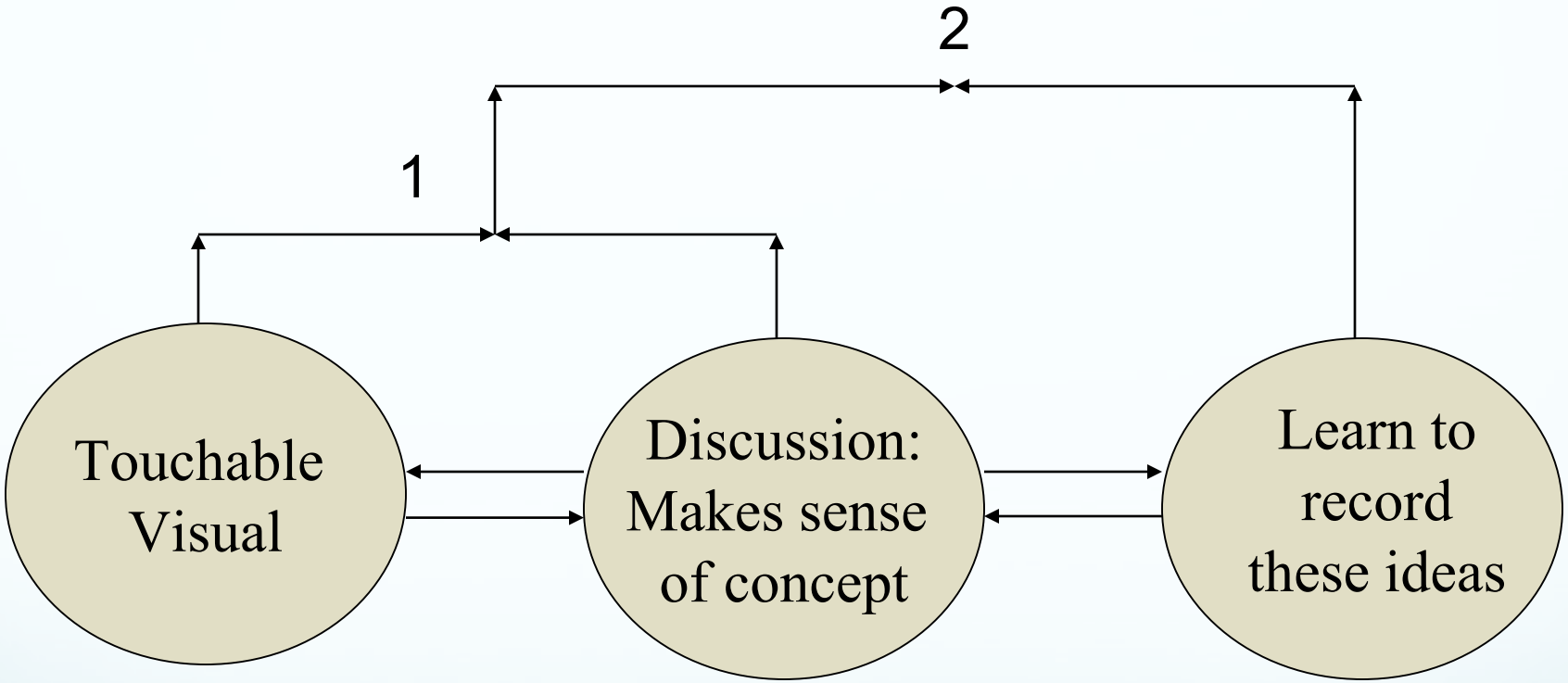
Know what you are teaching!

- 15-8 “Number Fact” (Automaticity)
- 15-8 Unlocking the Number System

Language Tips: Subtraction within 20

- Standard Form and Ones Form
- Equal refers to value
- A ten rod is not the same as ten ones
- Chinese language: “1 ten 2 ones”
- Mathematicians evaluate the form (e.g., “Is this the form I want my value in?”)

Prototype for Lesson Construction



Quantity

Concrete display of concept

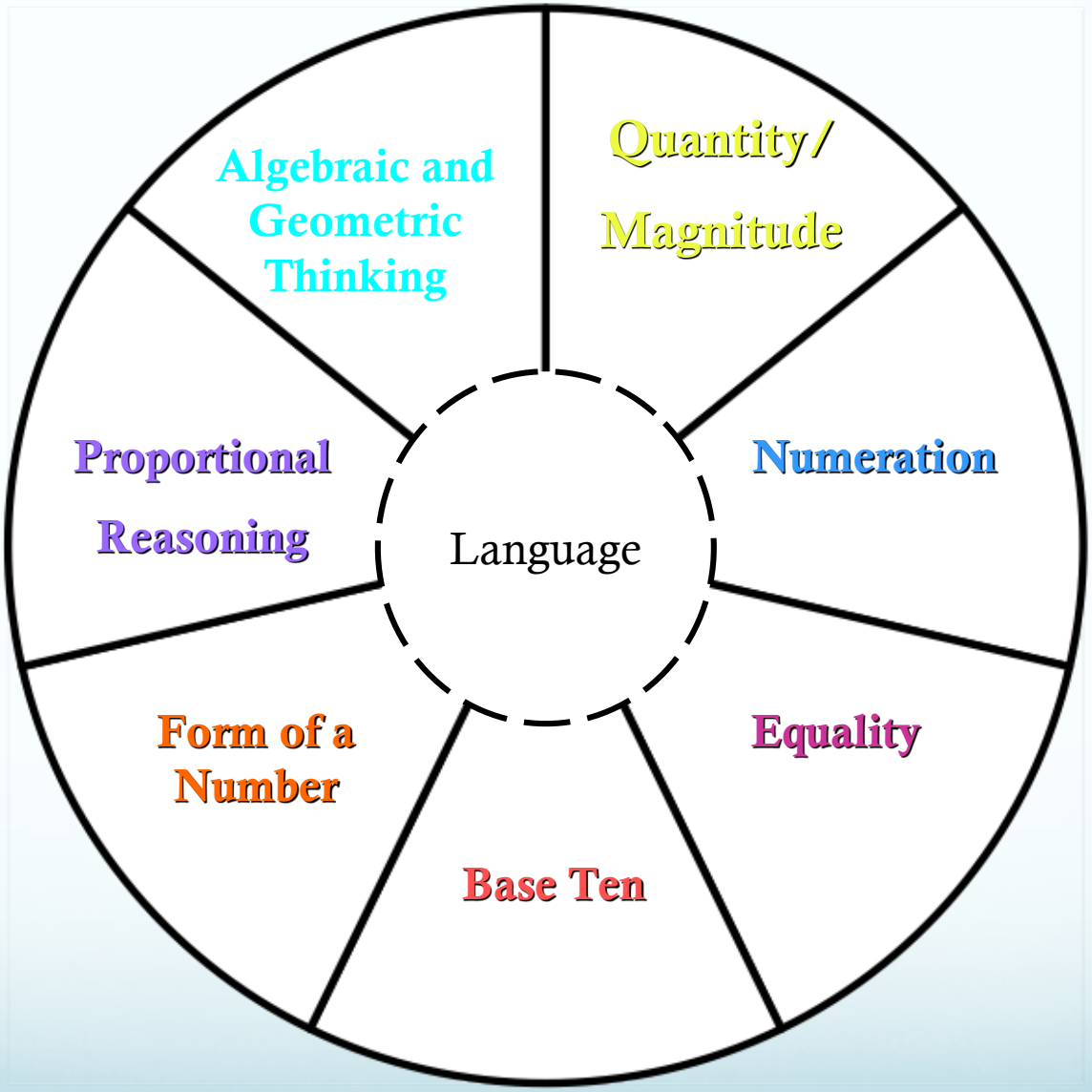
Mathematical Structure

Discussion of the concrete

Symbols

Simply record keeping!

V. Faulkner and DPI Task Force adapted from Griffin, 2003



Components of Number Sense

Language, Reading and Mathematics

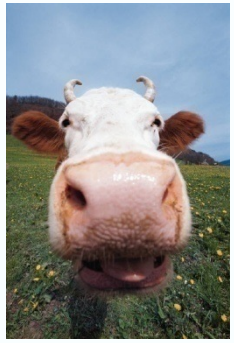
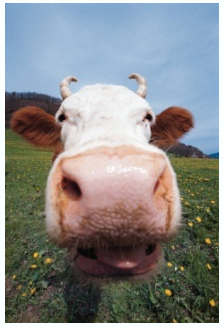
Connections and Disconnections

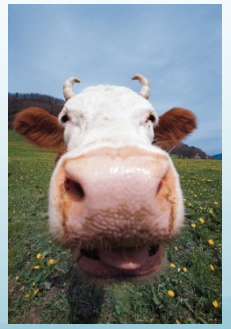
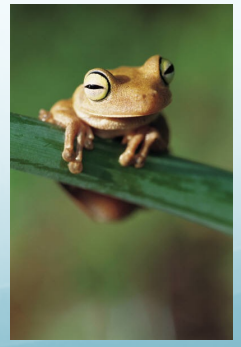
Connection to Categorizing

- 3 ones and 2 ones
- 3 tens and 2 tens
- 3 tens and 2 ones
- 3/6 and 2/6
- 3/6 and 2/5

Connection to Categorizing

- 3 ones and 2 ones
- 3 tens and 2 tens
- 3 tens and 2 ones
- 3/6 and 2/6
- 3/6 and 2/5

Concrete Reality

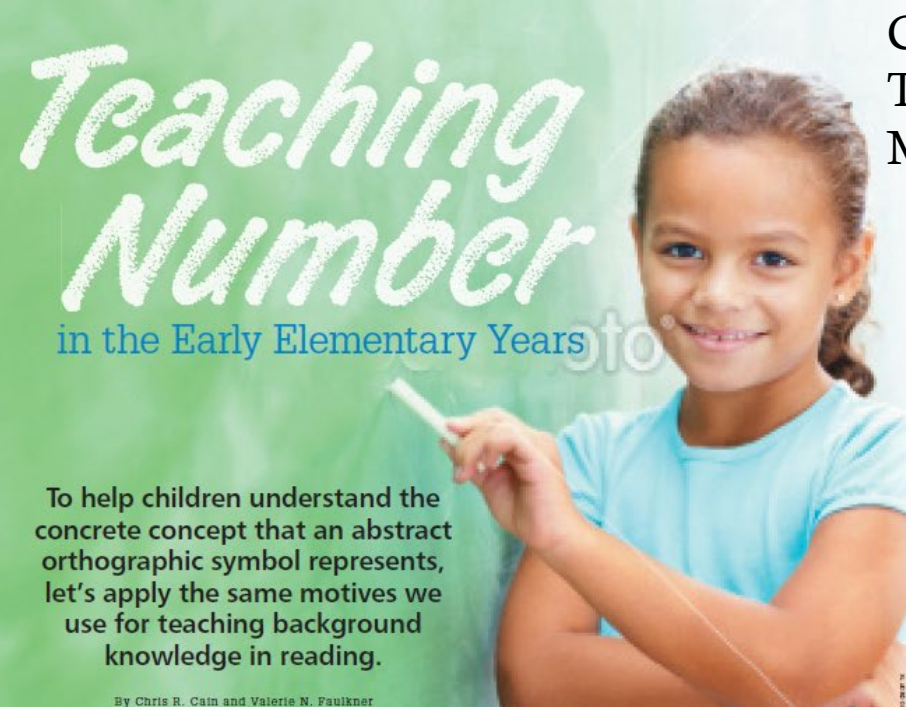
8 - 5 = 8

7 - 4 = 7

Cain, Faulkner in Teaching Children Mathematics

Teaching Number

in the Early Elementary Years



To help children understand the concrete concept that an abstract orthographic symbol represents, let's apply the same motives we use for teaching background knowledge in reading.

By Chris R. Cain and Valerie N. Faulkner

The widely adopted Common Core State Standards for Mathematics (CCSSI 2010) are designed to deepen instruction of number sense and will demand that elementary school teachers have a strong understanding of number. These changes arrive at a time when it is still understood that teachers and the curriculum in the United States have not been fundamentally driven by number sense connections (Ball and Cohen 1996). Teachers, therefore, are faced with the need to reflect on their own instructional choices and to make changes in their classrooms—changes that encourage the development of number sense in their students in keeping with the demands of the Common Core State Standards for Mathematics (CCSSM) and that go beyond what they have formerly thought about number (Ball and Cohen 1996). In our professional development with teachers from across our southeastern state, we have found that providing a model to develop the teacher's own sense of number is crucial. This model (see fig. 1) offers teachers an opportunity to reflect on their lessons and consider whether they have made mathematics connections that develop number sense in their students. By consciously exploring their own sense of number, teachers take an important step toward deepening their instruction in line with the CCSSM and creating classrooms that develop students' ability to reason abstractly and quantitatively, model situations with mathematics, and make use of mathematical structures.

Consistent with what we know about the importance of planning and reflection in lesson study (Hiebert and Stigler 2000; Stigler

How the Components of Number Sense Affected One Middle School Math Teacher

Dr. Chris Cain

As teacher educators, we have prioritized providing teachers with a tool that will substantially support their efforts to change their daily habits of language and instruction. We feel strongly that research must be made accessible to teachers so that they can effect change in their classrooms. It is our contention that this Model for Number Sense does just that.

One such example came in the college class, Advanced Methods of Mathematics Instruction. One of the participants in the class was a middle school teacher who had returned

to make the numeration system more clear to h
so she spoke to the class about equality and the
dents to tell her how these two forms of a number are equal. The class had a very hard time explaining the reason why the two forms of the number were equal.

Next, she had asked the class to use the blocks to show her 45%. She asked, "This is 45% of what?"; the class just looked at her. She explained that cent means 100 as in century and, therefore, percent means per 100. They were then able to articulate that 45% must be 45 out of 100. Then she

Faulkner in Teaching Exceptional Children

Designing Challenging Curriculum

The Components of Number Sense

An Instructional Model for Teachers

Valerie N. Faulkner

In recent years much attention has been placed on the relatively poor math performance of students in the United States (Gonzalez et al., 2004; Lemke et al., 2004; National Center for Education Statistics, 1999; National Research Council, 2001). Increased attention has also been paid to the struggling learner and mathematics. This includes issues regarding assessment (Gersten, Clarke, & Jordan, 2007); low-performing students in reform-based classrooms (Baxter, Woodward, & Olson, 2001); and general recommendations for the struggling student by the National Math Panel (Gersten et al., 2008).

The mathematical knowledge of teachers has also been investigated, and student success has been tied to the subtle factors of teacher implementation choices regarding problem sets, questioning techniques, and math connections (Hiebert & Stigler, 2000; Hill, Rowan, & Ball, 2005; Stigler & Hiebert,

Number Sense and Instructional Practice

At the heart of the recent focus on mathematics has been an increase emphasis on developing students' *number sense*. Ironically, although growing as a force in the educatio erature, number sense has not bee clearly defined for teachers.

Teachers need specific support understanding how to develop nu sense in students, to guide their le ing as they plan for and provide instruction (Ball & Cohen, 1996) a ultimately, to ensure that they are spending time encouraging studen do the thinking that will improve ber sense. A focus on content kno edge has been found to be an effe component of professional develop ment for teachers (Garet, Porter, Desimone, Birman, & Suk Yoon, 2 Hill et al., 2005), and teacher cont knowledge in mathematics has an impact on student performance (H

Cain in Teaching Exceptional Children

Are these the same?

$$4+4 = 7+1$$

$$1 = 1^3$$

Are these the same?

Equality and Form of a Number

$$7 = 3+4$$

Connections and Disconnections

- Oral Language
- Reading and Writing
- Mathematics

“This is all reading, when do we do the math EOG?”

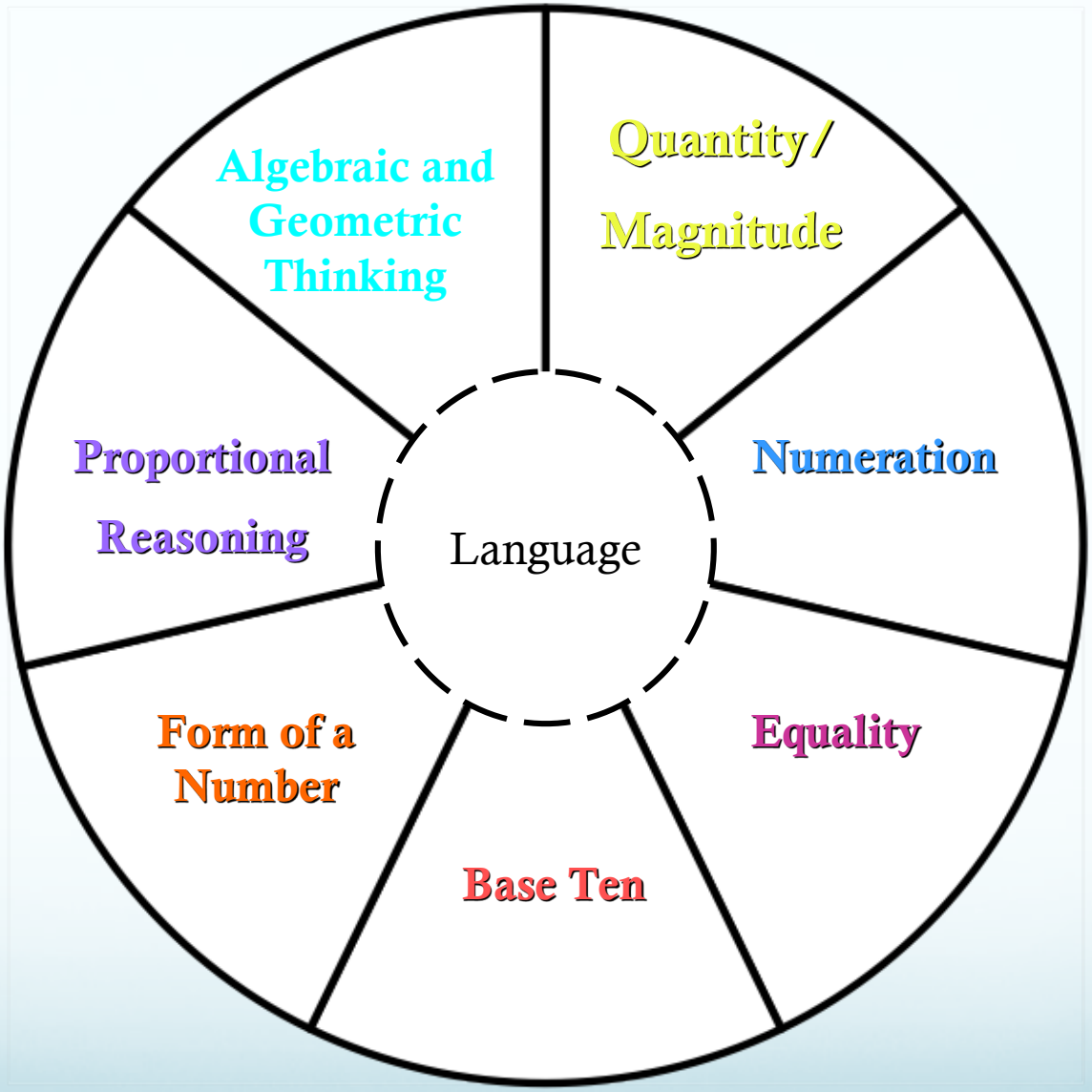
Key Words….

They don’t work!

We tell them—more means add

Erin has 46 comic books. She has 18 more comic books than Jason has. How many comic books does Jason have.

But is our answer really 64 which is 46 + 18?



Components of Number Sense

Universal Design for Learning Guidelines

Provide Multiple Means of Engagement

Purposeful, motivated learners

Provide options for self-regulation
- Promote expectations and beliefs that optimize motivation
- Facilitate personal coping skills and strategies
- Develop self-assessment and reflection

Provide options for sustaining effort and persistence
- Heighten salience of goals and objectives
- Vary demands and resources to optimize challenge
- Foster collaboration and community
- Increase mastery-oriented feedback

Provide options for recruiting interest
- Optimize individual choice and autonomy
- Optimize relevance, value, and authenticity
- Minimize threats and distractions

Provide Multiple Means of Representation

Resourceful, knowledgeable learners

Provide options for comprehension
- Activate or supply background knowledge
- Highlight patterns, critical features, big ideas, and relationships
- Guide information processing, visualization, and manipulation
- Maximize transfer and generalization

Provide options for language, mathematical expressions, and symbols
- Clarify vocabulary and symbols
- Clarify syntax and structure
- Support decoding of text, mathematical notation, and symbols
- Promote understanding across languages
- Illustrate through multiple media

Provide options for perception
- Offer ways of customizing the display of information
- Offer alternatives for auditory information
- Offer alternatives for visual information

Provide Multiple Means of Action & Expression

Strategic, goal-directed learners

Provide options for executive functions
- Guide appropriate goal-setting
- Support planning and strategy development
- Enhance capacity for monitoring progress

Provide options for expression and communication
- Use multiple media for communication
- Use multiple tools for construction and composition
- Build fluencies with graduated levels of support for practice and performance

Provide options for physical action
- Vary the methods for response and navigation
- Optimize access to tools and assistive technologies

“

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