Integrating the
8 Standards for Mathematical Practice
Into Your Daily Lessons

Part 2 - Standards 4, 5, 7, 8

Dr. Janet Stramel

Hello!

Fort Hays State University
- Mathematics Methods
- Concepts of Elementary Mathematics

National Board Certified Teacher
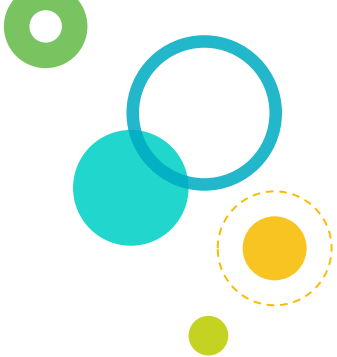- Early Adolescence Mathematics

Middle School Mathematics Teacher

I am Dr. Janet Stramel.

I am here because I love talking mathematics and love to give presentations.

You can find me at jkstramel@fhsu.edu

Session Outline

The Standards for Mathematical Practice reflect what it means to "do mathematics." This session will give you suggestions for incorporating them into your everyday mathematics instruction.

Standards for Mathematical Practice

Mathematically proficient students—

- **explain** to themselves the meaning of a problem and look for entry points to its solution.

- **make** sense of quantities and their relationships in problem situations.

- **use** assumptions, definitions, and previously established results in constructing arguments.

- **apply** the mathematics they know to solve problems arising in everyday life, society, and the workplace.

- **consider** all available tools when solving a mathematical problem.

- **communicate** precisely to others.

- **look** closely to discern a pattern or structure.

- **notice** if calculations are repeated and look for general methods and shortcuts.

Standards for Mathematical Practice

*Mathematically proficient students **do not** strictly...*

- **listen** while the teacher provides direct instruction for the entire class period.

- **copy** notes word for word from the board.

- **memorize** procedures for solving problems and duplicate exact replicas for homework and assessments.

- **sit** quietly throughout the entire lesson with minimal peer interaction.

Derek Pipkorn http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/What-Makes-a-Mathematically-Proficient-Student_/

CCSS Mathematical Practices

OVERARCHING HABITS OF MIND
1. Make sense of problems and persevere in solving them
6. Attend to precision

REASONING AND EXPLAINING
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS
4. Model with mathematics
5. Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Standards for Mathematical Practice

Standards 4 and 5

"Modeling and Using Tools"

SMP 4

Model with mathematics

➔ Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

SMP 4 -
Model with mathematics.

◎ Math limited to math class is worthless.
◎ Have students use math in science, art, music, and even reading.
◎ Use real graphics, articles, and data from the newspaper or other sources to make math relevant and real.
◎ Have students create real-world problems using their mathematical knowledge.

SMP 4 - Model with mathematics.

◎ **Elementary** - this might be as simple as writing an addition equation to describe a situation.

◎ **Middle Grades** - a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

◎ **High School** - a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

SMP 4 -
Model with mathematics

Teachers:

◎ Use mathematical models appropriate for the focus of the lesson
◎ Encourage student use of developmentally and content-appropriate mathematics models
◎ A mathematical model is a "work in progress"

Students:

◎ Apply prior knowledge
◎ Identify important quantities and map their relationships
◎ Make a problem simpler
◎ Does the answer make sense?

SMP 4 Sample Task

Analyzing Word Problems Involving Multiplication

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Many problems can be solved in different ways. Decide if the following word problems can be solved using multiplication. Explain your thinking. Then solve each problem.

a. Liam is cooking potatoes. The recipe says you need 5 minutes for every pound of potatoes you are cooking. How many minutes will it take for Liam to cook 12 pounds of potatoes?

b. Mel is designing cards. She has 4 different colors of paper and 7 different pictures she can glue on the paper. How many different card designs can she make using one color of paper and one picture?

c. Nina can practice a song 6 times in an hour. If she wants to practice the song 30 times before the recital, how many hours does she need to practice?

d. Owen is building a rectangular tile patio that is 4 tiles wide and 6 tiles long. How many tiles does he need?

SMP 4 Sample Task

From Illustrative Mathematics https://www.illustrativemathematics.org/content-standards/7/RP/A/tasks/114

Illustrative Mathematics

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Grade
7

Domain
Ratios And Proportional

Cluster
Analyze Proportional Relationships

**Task
Sale!**

Tags: **MP 4**

Alignments to Content Standards: **7.RP.A**

Student View

Task

Four different stores are having a sale. The signs below show the discounts available at each of the four stores.

| Two for the price of one | Buy one and get 25% off the second |
| Buy two and get 50% off the second one | Three for the price of two |

a. Which of these four different offers gives the biggest price reduction? Explain your reasoning clearly.

b. Which of these four different offers gives the smallest price reduction? Explain your reasoning clearly.

SMP 4 - Questions to Develop Mathematical Thinking
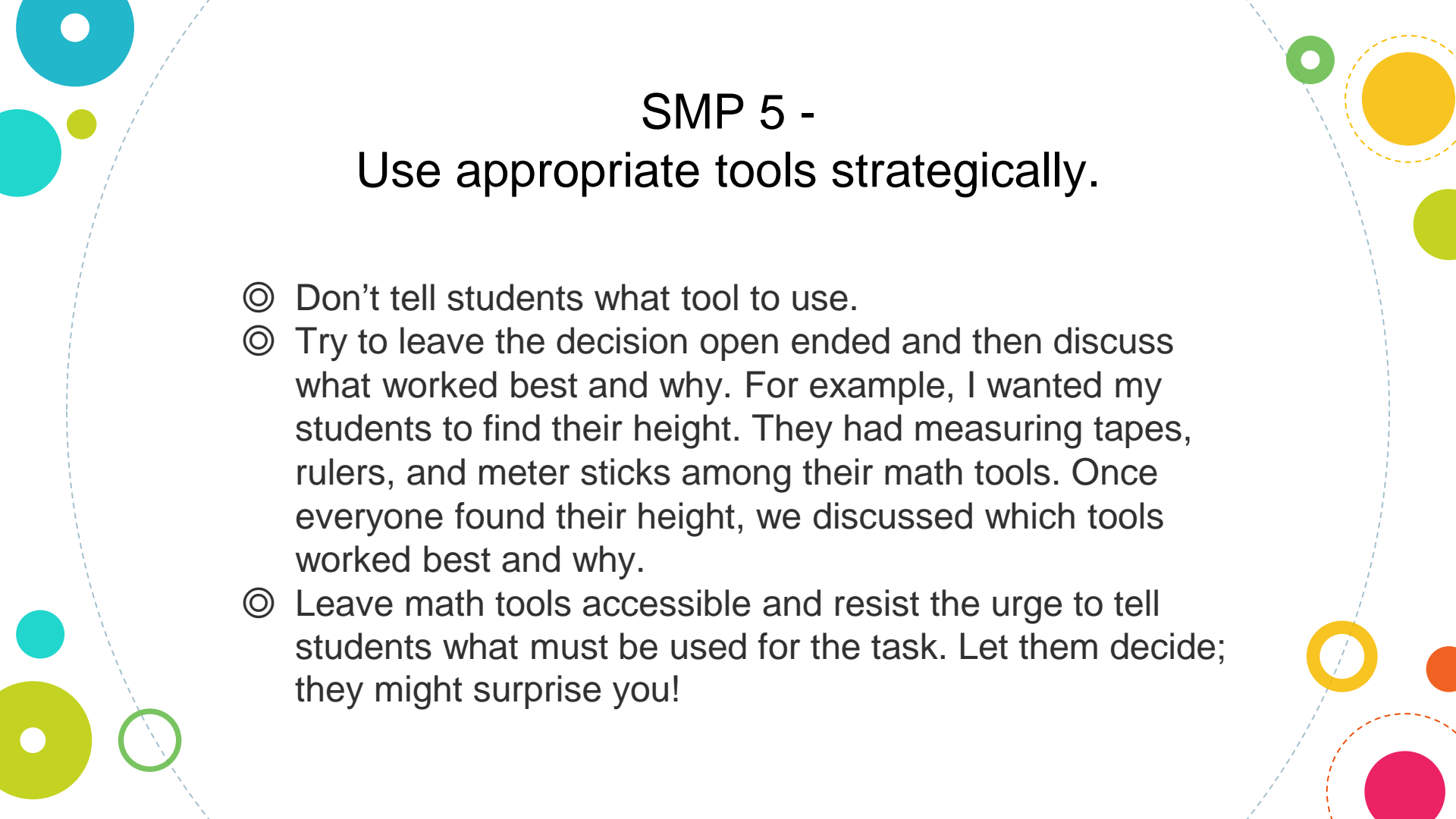
◎ What number model could you construct to represent the problem?

◎ What are some ways to represent the quantities?

◎ What is an equation or expression that matches the diagram..., number line.., chart..., table..?

◎ Would it help to create a diagram, graph, table...?

◎ What are some ways to visually represent...?

◎ What formula might apply in this situation?

SMP 5
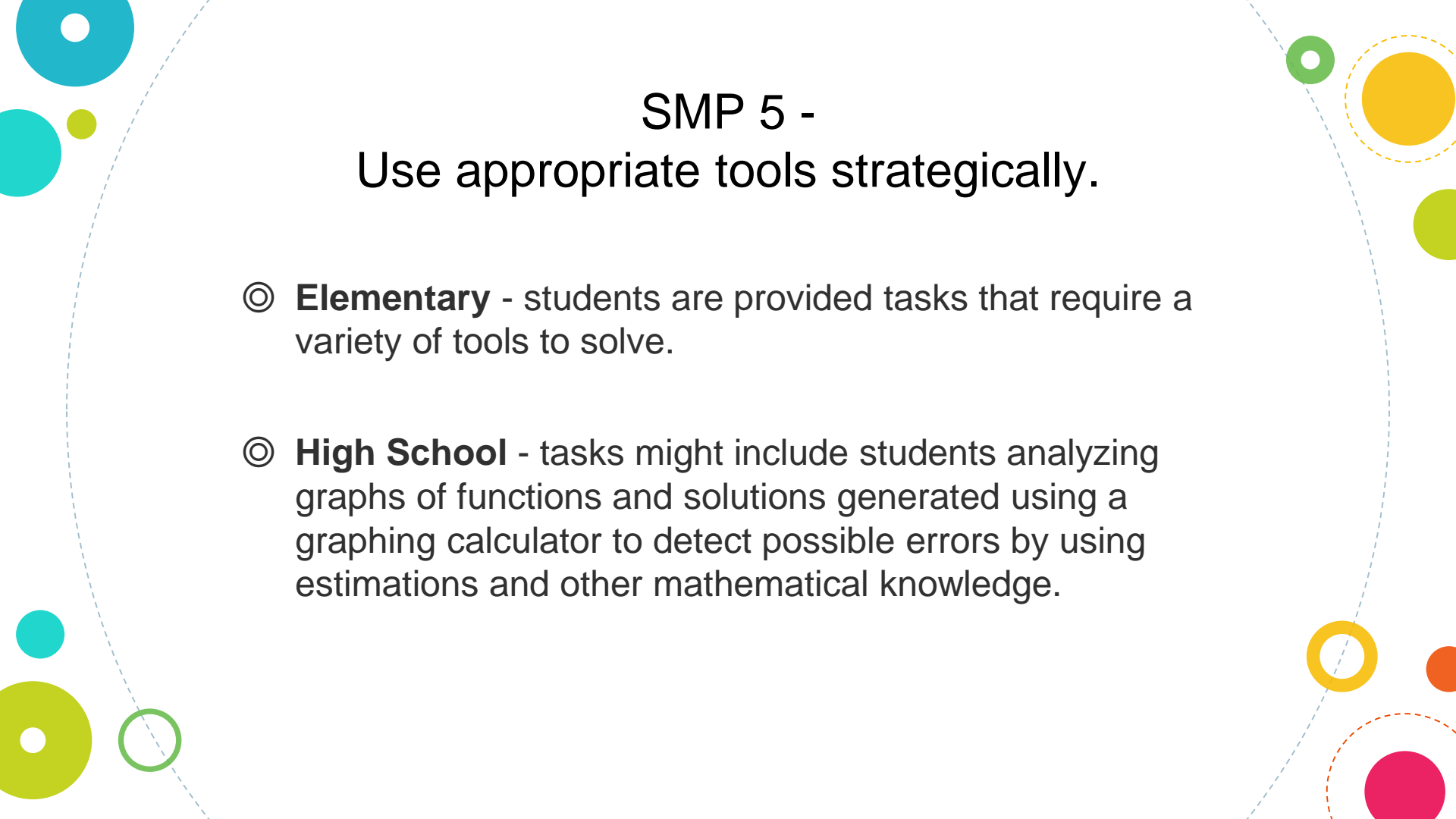
Use appropriate tools strategically

➜ Mathematically proficient students consider all available tools when solving a mathematical problem.

SMP 5 -
Use appropriate tools strategically.

◎ Don't tell students what tool to use.
◎ Try to leave the decision open ended and then discuss what worked best and why. For example, I wanted my students to find their height. They had measuring tapes, rulers, and meter sticks among their math tools. Once everyone found their height, we discussed which tools worked best and why.
◎ Leave math tools accessible and resist the urge to tell students what must be used for the task. Let them decide; they might surprise you!

SMP 5 -
Use appropriate tools strategically.

◎ **Elementary** - students are provided tasks that require a variety of tools to solve.

◎ **High School** - tasks might include students analyzing graphs of functions and solutions generated using a graphing calculator to detect possible errors by using estimations and other mathematical knowledge.

SMP 5 -
Use appropriate tools strategically

Teachers:

◎ Provides a variety of tools and technology for students to explore to deepen their understanding of math concepts

◎ Provides problem solving tasks that require students to consider a variety of tools for solving

Students:

◎ Consider available tools when solving a mathematical problem

◎ Are familiar with a variety of mathematics tools

◎ Use technological tools to explore and deepen understanding of concepts

SMP 5 Choices

	K–2	3–5	6–8
Number Sense	**Adding Whole Numbers** Teddy Bear Counters Number Line Pen and Pencil Hundreds Grids Number Bonds	**Adding Fractions** Number Lines Unit Circles Rectangular Area Models Pen and Pencil Benchmarks	**Adding Integers** Number Line Two-Colored Counters Flashcards Deck of Cards (Red/Black)
Algebra	**Repeating Patterns** Crayons and Paper Multilink Cubes Colored Counters or Beans Pattern Blocks	**Solve One Unknown Problem** Tape Diagrams Open Sentences Variable Equations Picture Representations	**Solving Equations** Algebra Tiles Pan Balances Algebraic Properties Pencil and Paper
Geometry	**Describing Figures** Real Life Objects Pattern Blocks Toothpicks or Straws Whiteboards	**Properties of Quadrilaterals** Coordinate Grids Real Life Objects Ruler and Protractor Transparencies and Markers	**Geometric Transformations** Transparencies and Markers Tracing Paper Coordinate Grids Compass and Straightedge
Data	**Organizing Data** Line Plots Picture Graphs Counters Bar Graphs T-Charts	**Creating Scales for Data** Number Line Grid Paper Ruler and Blank Paper Skip Counting	**Analyze Center and Variability** Line Plot Calculator Spreadsheet Histogram Box and Whiskers Plot

SMP 5 - Questions to Develop Mathematical Thinking

◎ What mathematical tools could we use to visualize and represent the situation?

◎ What do you know that is not stated in the problem?

◎ What approach are you considering trying first?

◎ What estimate did you make for the solution?

◎ In this situation would it be helpful to use...a graph...,number line..., ruler..., diagram...,

Standards for Mathematical Practice

Standards 7 and 8

"Seeing Structure and Generalizing"

SMP 7

Look for and make use of structure

➔ Mathematically proficient students look closely to discern a pattern or structure.

SMP 7 -
Look for and make use of structure.

◎ Help students identify multiple strategies and then select the best one.

◎ Repeatedly break apart numbers and problems into different parts.

◎ Use what you know is true to solve a new problem.

◎ Prove solutions without relying on the algorithm. For example, my students are changing mixed numbers into improper fractions. They have to prove to me that they have the right answer without using the "steps."

SMP 7 -
Look for and make use of structure.

◎ **Elementary** - Tasks might require students to notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.

◎ **Middle School and High School** - Students can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
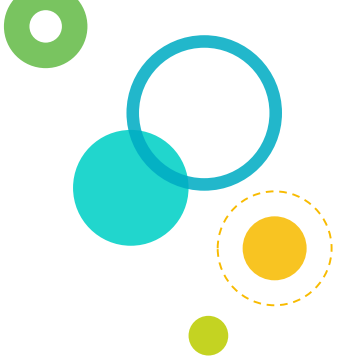
SMP 7 -
Look for and make use of structure

Teachers:

◎ Provides opportunities and time for students to explore patterns and relationships
◎ Provides rich tasks and facilitates pattern seeking and understanding of relationships
◎ Engages students in discussions
◎ Recognizes that quantitative relationships are modeled by operations and their properties remain important regardless of the operational focus of a lesson
◎ Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways

Students:

◎ Discern patterns or structure
◎ Associate patterns with properties of operations and their relationships
◎ See complicated things as single objects or as composed of several objects.
◎ Look for patterns or structure
◎ Recognize the significance in concepts and models and use the patterns or structure for solving related problems
◎ View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems

SMP 7 Sample Task

Example problems from Gail Burrill:

- Solve for x: 3(x − 2) = 9

Rather than approach the problem above by distributing or dividing, a student who uses structure would identify that the equation is saying 3 times something is 9 and thus the quantity in parentheses must be 3. Therefore, x = 5.

- Solve for x: $\dfrac{3}{x-1} = \dfrac{6}{x+3}$

The "typical" approach to the above problem would be to cross multiply and solve; a student who identifies and makes use of structure sees that the left side can be multiplied by 2 to create equivalent numerators… then simply set the denominators equal and solve.

SMP 7 - Questions to Develop Mathematical Thinking

◎ What observations do you make about...?

◎ What do you notice when...?

◎ What patterns do you find in...?

◎ How do you know if something is a pattern?

◎ What ideas that we have learned before were useful in solving this problem?

◎ What are some other problems that are similar to this one?

SMP 8

Look for and express regularity in repeated reasoning

➔ Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts

SMP 8 -
Look for and express regularity in repeated reasoning.

◎ Show students how the problem works. As soon as they "get it," start making them generalize to a variety of problems.

◎ Don't work fifty of the same problem; take your mathematical reasoning and apply it to other situations.

SMP 8 -
Look for and express regularity in repeated reasoning.

◎ **Elementary** - solving problems and noticing that when dividing 25 by 11 they are repeating the same calculations over and over again, and conclude they have a repeating decimal.

◎ **Upper elementary** - students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal.

◎ **Middle School** - By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, students might abstract the equation $(y-2)/(x-1)=3$.

SMP 8 -
Look for and express regularity in repeated reasoning

Teachers:

◎ Provides problem situations that allow students to explore regularity and repeated reasoning

◎ Provides rich tasks that encourage students to use repeated reasoning to form generalizations and provides opportunities for students to communicate these generalizations

◎ Engages students in discussion

◎ Urge students to continually evaluate the reasonableness of their results

Students:

◎ Notice if calculations are repeated and look for both general methods and shortcuts

◎ Use regularity and use this to lead to a general formula and generalizations

◎ Maintain oversight of the process of solving a problem while attending to details and continually evaluates the reasonableness of immediate results.

◎ Notice repeated calculations and look for general methods and shortcuts

◎ Continually evaluate the reasonableness of intermediate results

SMP 8 Task

From Illustrative Mathematics

- 4.OA Double Plus One
- 6.NS Adding Multiples*
- 6.EE The Djinni's Offer
- 8.EE Extending the Definitions of Exponents
- F-LE Equal Differences over Equal Intervals

SMP 8 Sample Task

https://www.illustrativemathematics.org/content-standards/6/NS/B/4/tasks/257

Grade 6	
Domain The Number System	
Cluster Compute Fluently With Multi-Digit	
Standard Find The Greatest Common Factor	
Task Adding Multiples	

Adding Multiples

No Tags

Alignments to Content Standards: 6.NS.B.4

Student View

Task

Nina was finding multiples of 6. She said,

18 and 42 are both multiples of 6, and when I add them, I also get a multiple of 6:

$$18 + 42 = 60.$$
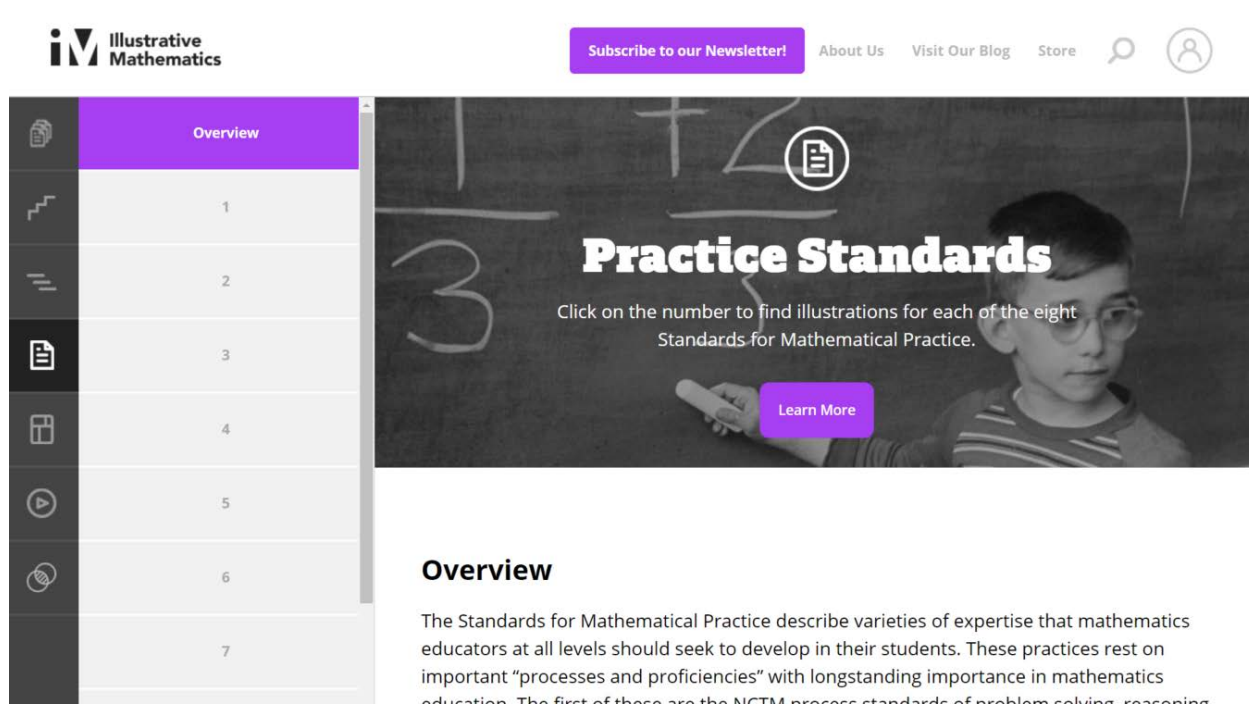
Explain to Nina why adding two multiples of 6 will always result in another multiple of 6.

SMP 8 - Questions to Develop Mathematical Thinking

◎ Will the same strategy work in other situations?

◎ Is this always true, sometimes true or never true? Use evidence to explain.

◎ How would we prove that...?

◎ What do you notice about...?

◎ Is there a mathematical rule for...? Explain the rule and where it came from?

Illustrative Mathematics:
Practice Standards

Mathematics Assessment Project

http://map.mathshell.org/stds.php?standardid=1159

Other Resources of Note:

◎ Kansas FlipBooks
- ◉ http://community.ksde.org/Default.aspx?tabid=5646

◎ Blog by Bill McCallum
- ◉ http://commoncoretools.me/

◎ You Cubed
- ◉ https://www.youcubed.org/

◎ Debbie Waggoner
- ◉ http://www.debbiewaggoner.com/math-practice-standards.html

◎ Inside Mathematics
- ◉ http://www.insidemathematics.org/

Thank You!

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Any questions?

You can find me at jkstramel@fhsu.edu

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