Making Sense of the Math Through Fractions

2020 - 1-hour presentation

Equals Mathematics

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PISA: Context

Program for International Student Assessment

- USA ranked third in the OECD sample in per capita GDP
- USA ranked fourth in the OECD sample in per student spending
- The share of students from disadvantaged backgrounds is within the average range of the OECD sample

OECD, 2014

PISA: Results

Program for International Student Assessment

The USA average score was 27th out of 34 countries*

26% of USA students scored below the baseline level of proficiency

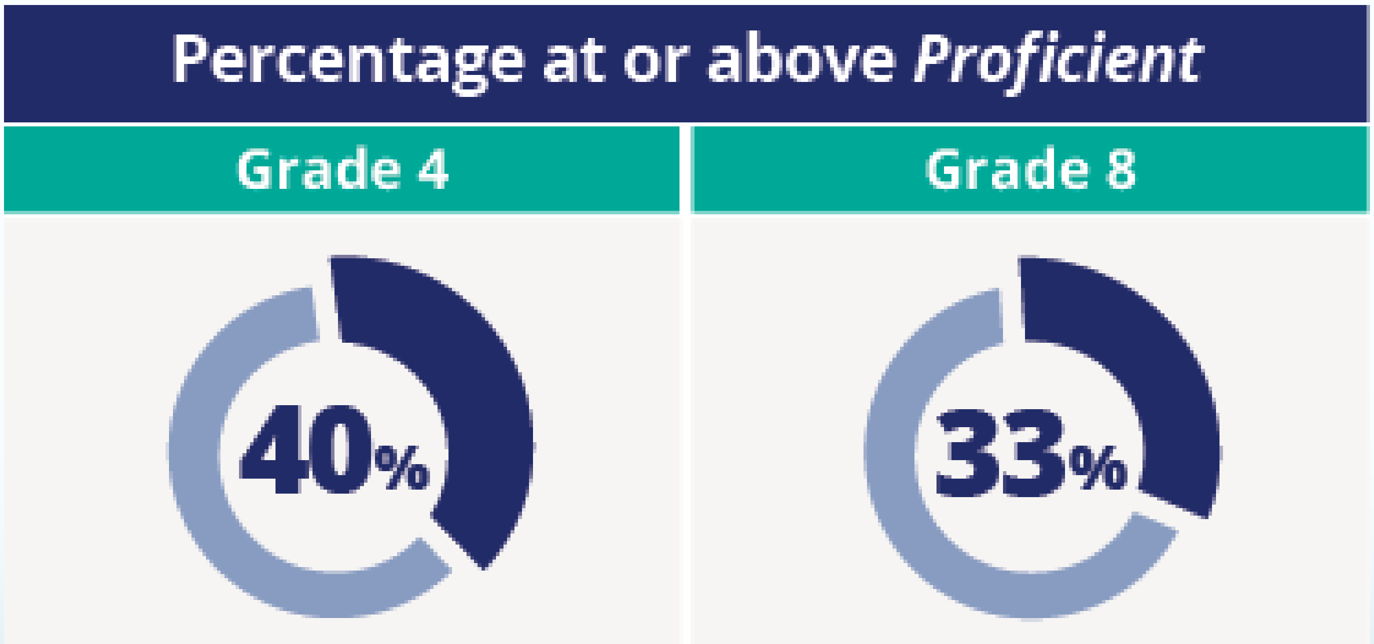
9% of USA students scored within the top proficiency level

*of OECD participating countries

OECD, 2014

NAEP: 2015

National Assessment of Educational Progress



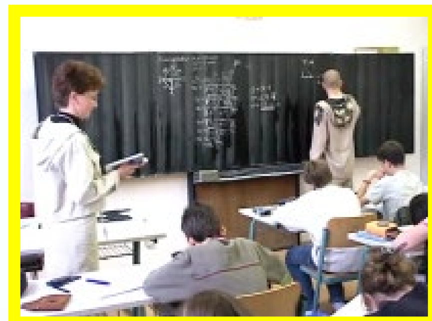
National Center for Educational Statistics, 2015

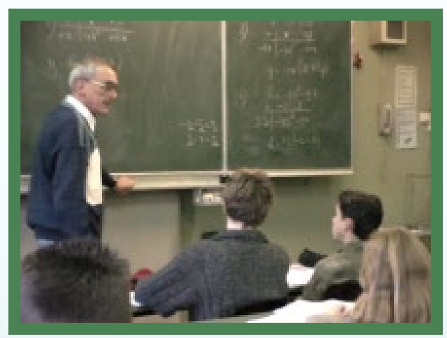
TIMSS

Trends in International Mathematics and Science Study

US Teachers		Hong Kong, Singapore, Japan
Learning terms and practicing procedures	Instructional focus	Structured problem solving
Covers 80% of tested topics	Pace	About half the tested topics
Mile wide, inch deep	Curriculum	Greater depth and coherence
How can I teach my students to get the answer to this problem?	Teachers plan by asking…	How can I use this exercise to teach mathematics they don't already know?

International Research


TIMSS Video Studies

- 1995 Video Study
  - Japan, Germany, US
  - Teaching Style Implicated
- 1999 Video Study
  - US, Japan, Netherlands, Hong Kong, Australia, Czech Rep.
  - Implementation Implicated

Stigler & Hiebert, 2004

Workspace

High Achieving Countries

MAKE CONNECTIONS

United States

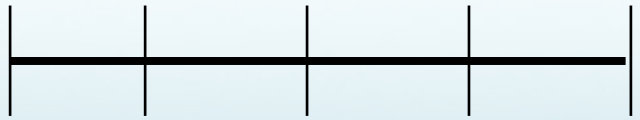
TEACHES PROCEDURES

Structures and Connections

What is 4^2 ?

Procedure versus Structure/Connections

Make a square out of your 4 unit linear side



Workspace

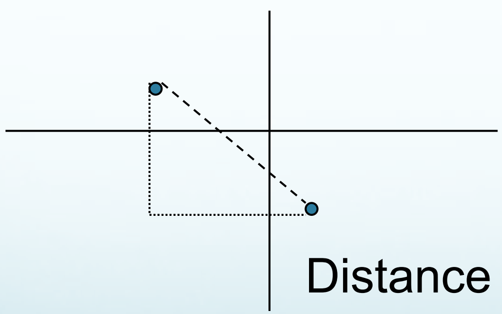
Exponents and CONNECTIONS

Square Roots!

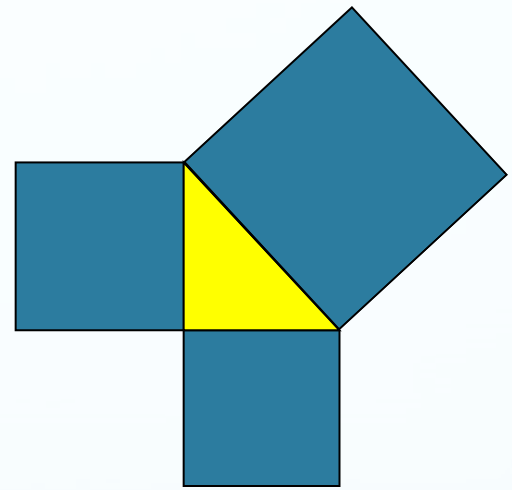
$\sqrt{16} = 4$

The length of one side!

Geometry and Measurement



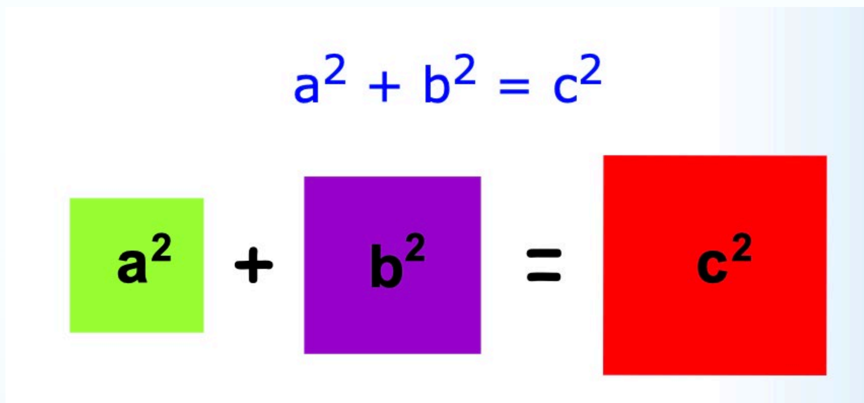
Distance Formula



Pythagorean
Special Triangles
Trigonometry

Connections

4^2



$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Understanding and Instruction

We can only instruct our students as well as we understand the mathematics:

The better we understand the math, the better decisions we will make regarding what the student needs to achieve and how to instruct the student!

Defining Issue in Implementation

…is the teacher's <u>own</u> understanding of Mathematics.

Liping Ma (1999)

What does it mean to divide

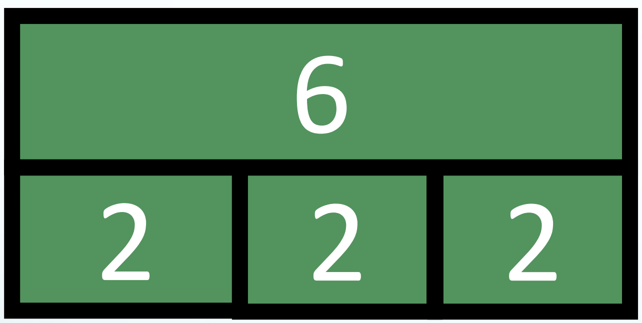
$$6/2=?$$

Consider 6÷2…

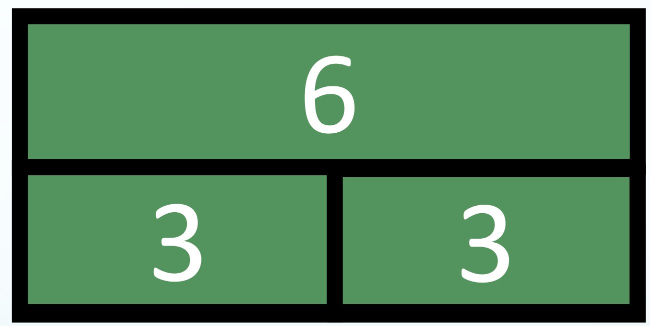
26 - 27

$$\frac{6 \text{ cups}}{2 \text{ cups}} = 3 \text{ servings}$$

$$\frac{6 \text{ cups}}{2 \text{ servings}} = \frac{3 \text{ cups}}{\text{serving}}$$



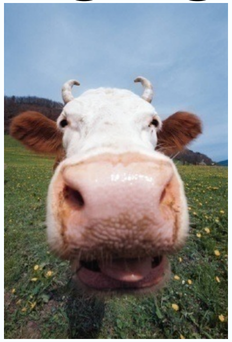
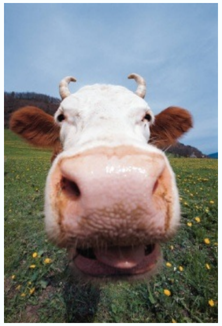
Repeated Subtraction

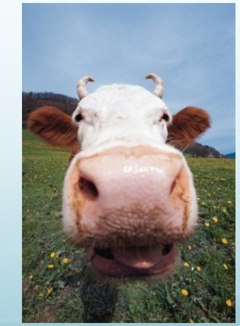


Partitive/Unit Rate

Connection to Language / Units

- 3 ones and 2 ones
- 3 tens and 2 tens
- 3 tens and 2 ones
- 3/6   and 2/6
- 3/6 and 2/5


Knowing and Teaching Elementary Mathematics (Liping Ma)

- Compared and contrasted the pedagogy of Chinese and American teachers
- Found that American teachers were much weaker in content and conceptual knowledge
- Found American teachers teach procedurally rather than being driven by the logic of the mathematics (implementation)
- Ma presented information through teacher responses to elementary math questions

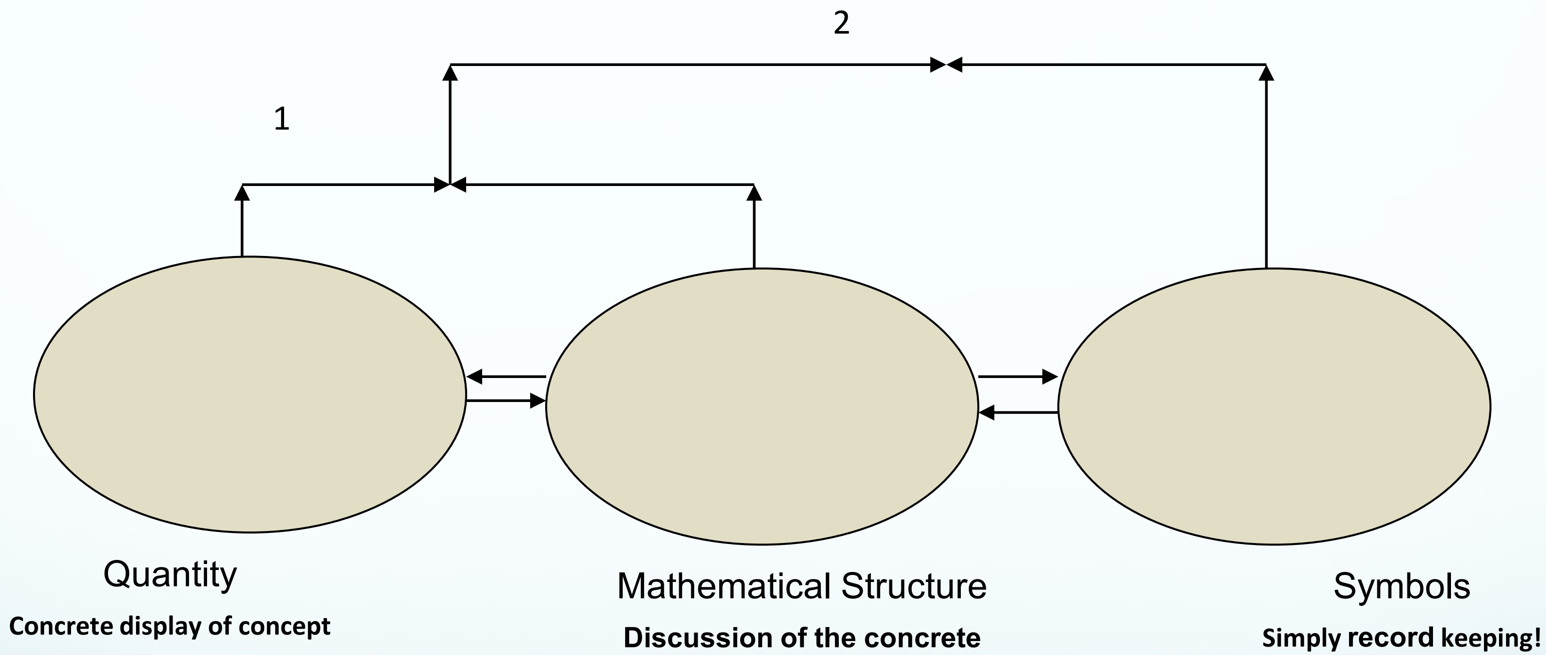
Problem #3 Division of Fractions

1 ¾ divided by ½

Give a Story Problem to show what is happening with this expression.

Ma, 1999

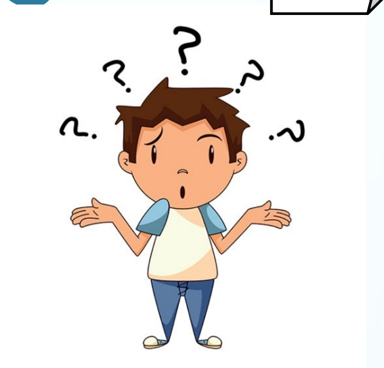
Division of Fractions Lesson Construction



V. Faulkner and NCDPI Task Force adapted from Griffin

Division of Fractions

17



- **American teachers' approach**
  - Flip and multiply
  - Answers don't match
  - Confused division by two with division by one-half

Ma, 1999

Division of Fractions

18

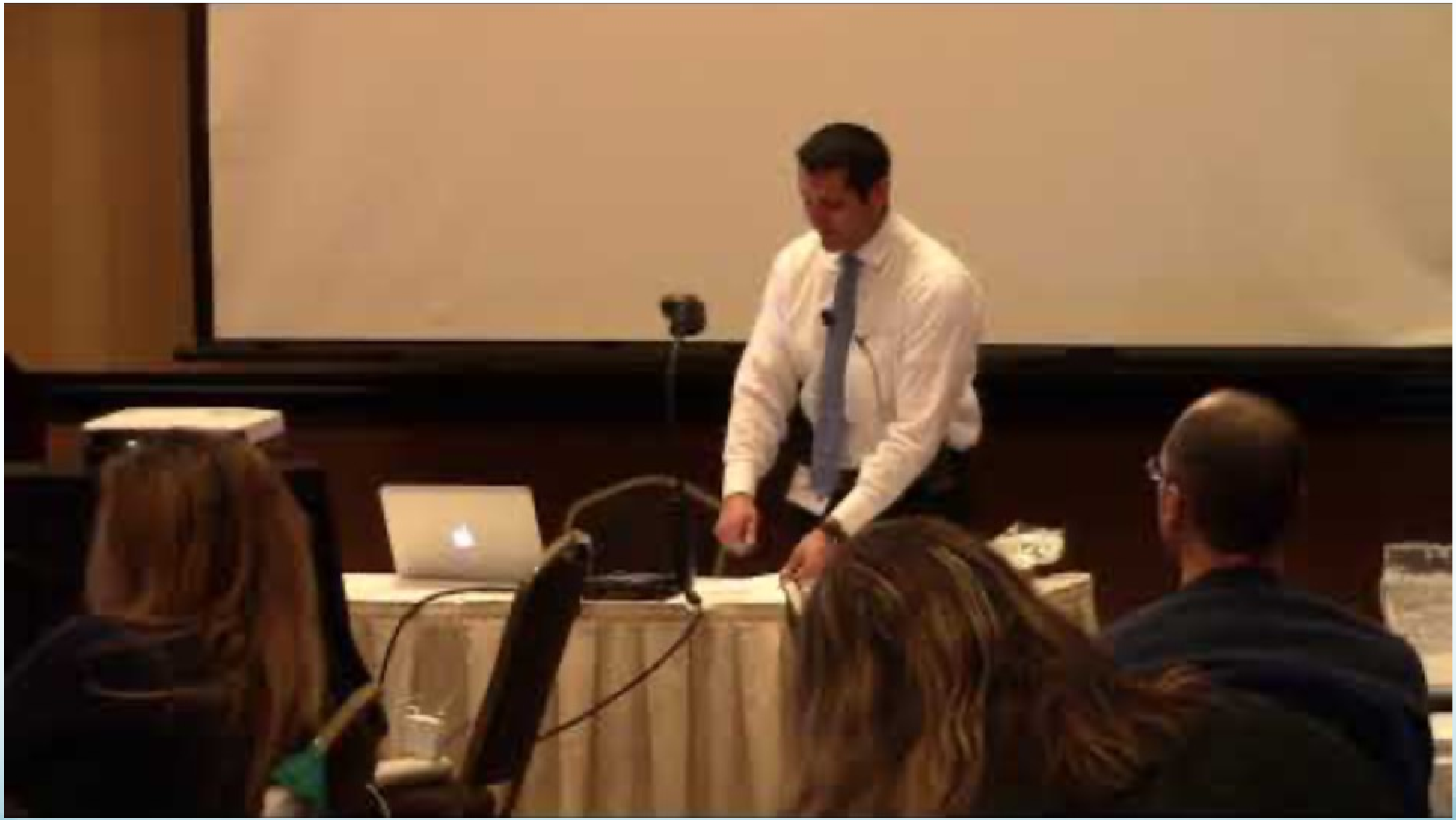
- **Chinese teachers' approach**
  - Gave a mathematically accurate story problem
  - Explained the mathematics behind the operation
  - Gave multiple mathematical constructs for division of fractions

Ma, 1999

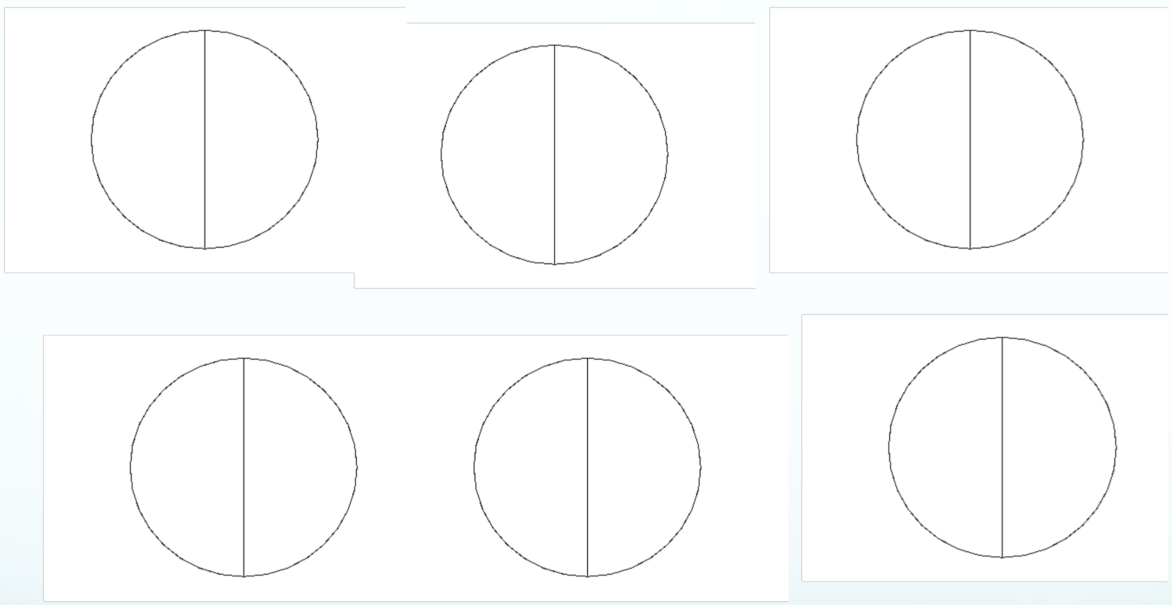
Division of Fractions

What does it mean to divide by a fraction?

Movie Time



For each Cup of Beans—
I get two portions if I divide by 1/2



Workspace

What if I divide by 1/4?

- How would my diagram look?
- How many portions would I have?
- Why are my number of portions getting larger?
- How does this idea of quantity tie into the math structure of proportional reasoning and repeated subtraction?

Bean Party #2

Serving Size: 1/2 cup of Beans

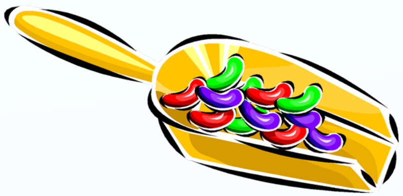
How many servings can be made from:

- 1 cup of beans
- 2 cups of beans
- 3 1/2 cups of beans

Bean Party #3

How many 2 cup servings can be made from:

- 8 cups of beans
- 5 cups of beans
  - Which one is harder for students?
  - What would the mathematical sentence look like?



What About Leftover Beans?

8 cups of beans ÷ 2 cup servings = 4 servings of 2 cups each

9 cups of beans ÷ 2 cup servings ≠ 4 servings remainder 1

Leftover Beans

9 cups of beans ÷ 2 cup servings =

4 servings of 2 cups AND 1 cup of a 2 cup serving

Leftover Beans

9 cups of beans ÷ 2 cup servings =

4 ½ groups of 2 or 4 ½ servings

Division of Fractions: Bean Party!
Division as Repeated Subtraction

2

1

"Why does this constant amount of beans "looks like it is getting bigger"? Division as repeated subtraction

2 ½ / ½

Quantity
Concrete display of concept

Mathematical Structure
Discussion of the concrete

Symbols
Simply record keeping!

V. Faulkner and DPI Task Force adapted from Griffin

Cain, Faulkner in Teaching Children Mathematics

Teaching Number

in the Early Elementary Years

To help children understand the concrete concept that an abstract orthographic symbol represents, let's apply the same motives we use for teaching background knowledge in reading.

By Chris R. Cain and Valerie N. Faulkner

The widely adopted Common Core State Standards for Mathematics (CCSSI 2010) are designed to deepen instruction of number sense and will demand that elementary school teachers have a strong understanding of number. These changes arrive at a time when it is still understood that teachers and the curriculum in the United States have not been fundamentally driven by number sense connections (Ball and Cohen 1996). Teachers, therefore, are faced with the need to reflect on their own instructional choices and to make changes in their classrooms—changes that encourage the development of number sense in their students in keeping with the demands of the Common Core State Standards for Mathematics (CCSSM) and that go beyond what they have formerly thought about number (Ball and Cohen 1996). In our professional development with teachers from across our southeastern state, we have found that providing a model to develop the teacher's own sense of number is crucial. This model (see fig. 1) offers teachers an opportunity to reflect on their lessons and consider whether they have made mathematics connections that develop number sense in their students. By consciously exploring their own sense of number, teachers take an important step toward deepening their instruction in line with the CCSSM and creating classrooms that develop students' ability to reason abstractly and quantitatively, model situations with mathematics, and make use of mathematical structures.

Consistent with what we know about the importance of planning and reflection in lesson study (Hiebert and Stigler 2000; Stigler

Faulkner in Teaching Exceptional Children

Designing Challenging Curriculum

The Components of Number Sense

An Instructional Model for Teachers

Valerie N. Faulkner

In recent years much attention has been placed on the relatively poor math performance of students in the United States (Gonzalez et al., 2004; Lemke et al., 2004; National Center for Education Statistics, 1999; National Research Council, 2001). Increased attention has also been paid to the struggling learner and mathematics. This includes issues regarding assessment (Gersten, Clarke, & Jordan, 2007); low-performing students in reform-based classrooms (Baxter, Woodward, & Olson, 2001); and general recommendations for the struggling student by the National Math Panel (Gersten et al., 2008).

The mathematical knowledge of teachers has also been investigated, and student success has been tied to the subtle factors of teacher implementation choices regarding problem sets, questioning techniques, and math connections (Hiebert & Stigler, 2000; Hill, Rowan, & Ball, 2005; Stigler & Hiebert,

Number Sense and Instructional Practice

At the heart of the recent focus on mathematics has been an increase emphasis on developing students' *number sense.* Ironically, although growing as a force in the education erature, number sense has not bee clearly defined for teachers.

Teachers need specific support understanding how to develop nu sense in students, to guide their le ing as they plan for and provide instruction (Ball & Cohen, 1996) a ultimately, to ensure that they are spending time encouraging studen do the thinking that will improve ber sense. A focus on content kno edge has been found to be an effe component of professional develop ment for teachers (Garet, Porter, Desimone, Birman, & Suk Yoon, 2 Hill et al., 2005), and teacher cont knowledge in mathematics has an impact on student performance (H

How the Components of Number Sense Affected One Middle School Math Teacher

Dr. Chris Cain

As teacher educators, we have prioritized providing teachers with a tool that will substantially support their efforts to change their daily habits of language and instruction. We feel strongly that research must be made accessible to teachers so that they can effect change in their classrooms. It is our contention that this Model for Number Sense does just that.

One such example came in the college class, Advanced Methods of Mathematics Instruction. One of the participants in the class was a middle school teacher who had returned

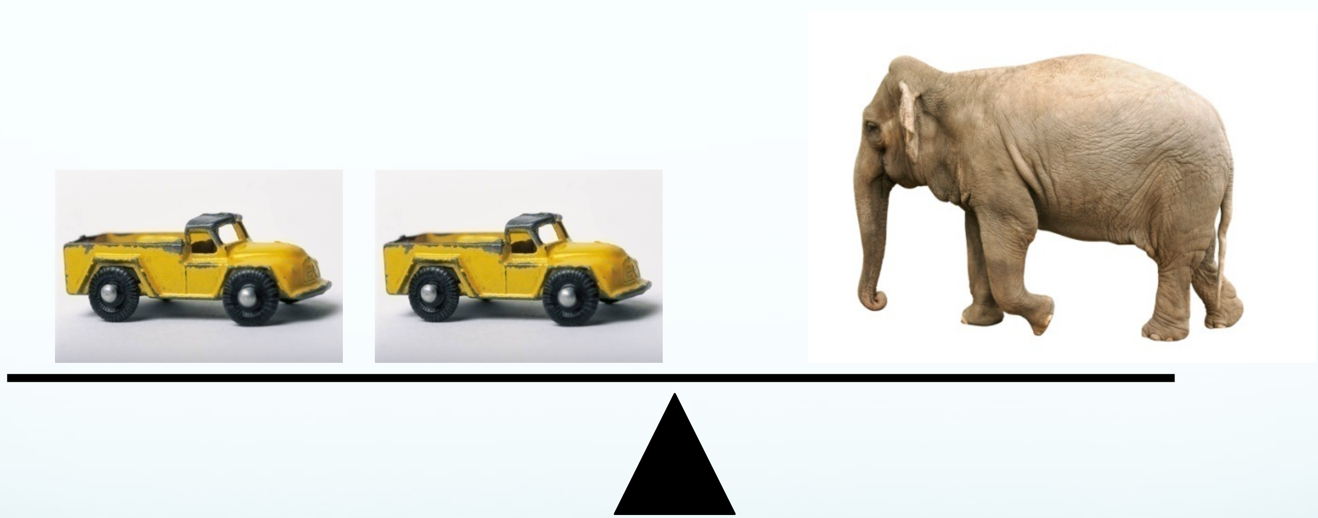
to make the numeration system more clear to h so she spoke to the class about equality and the dents to tell her how these two forms of a number are equal. The class had a very hard time explaining the reason why the two forms of the number were equal.

Next, she had asked the class to use the blocks to show her 45%. She asked, "This is 45% of what?"; the class just looked at her. She explained that cent means 100 as in century and, therefore, percent means per 100. They were then able to articulate that 45% must be 45 out of 100. Then she

Cain in Teaching Exceptional Children

Are these the same?

e same?

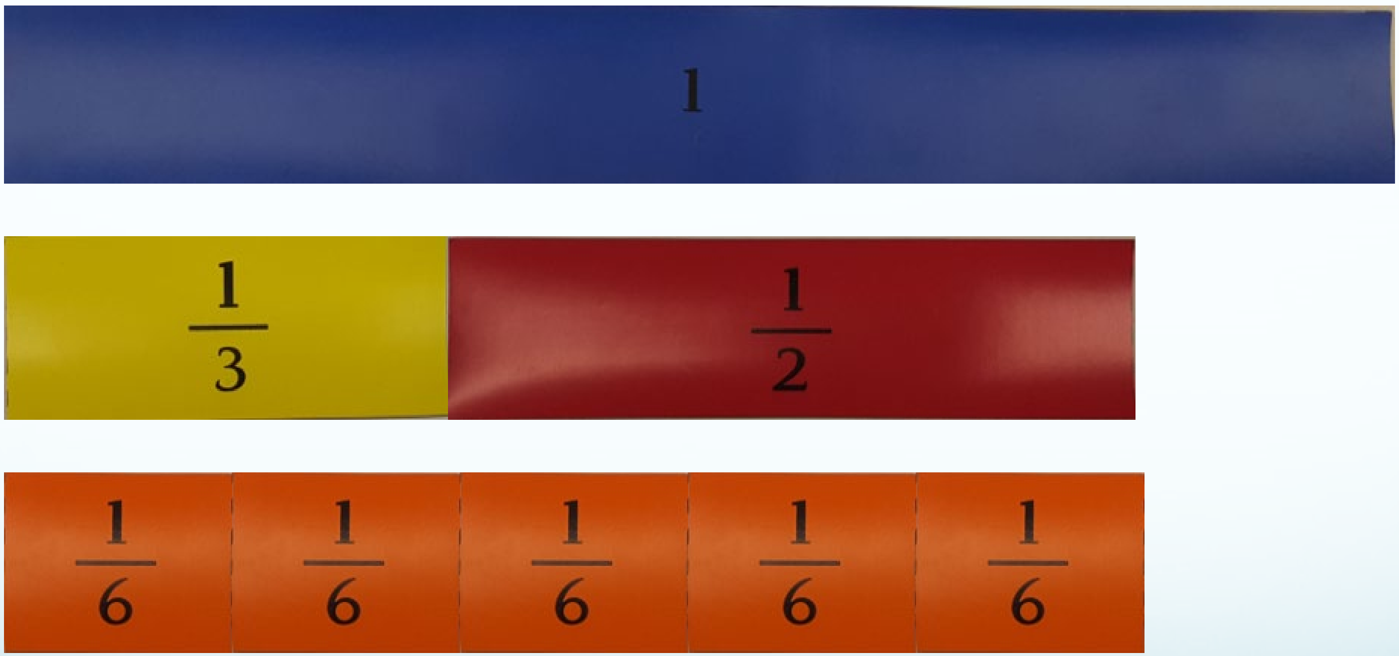


Equality and Form of a Number

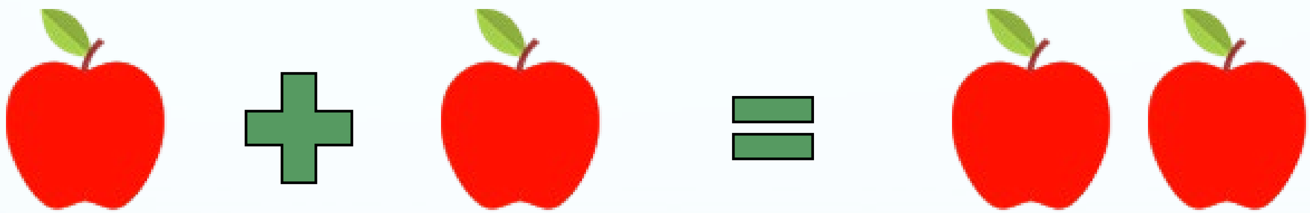
$$1/2 = 3/6$$

Pumpkin Pie

Computation with Fractions: Modeling with Fraction Strips

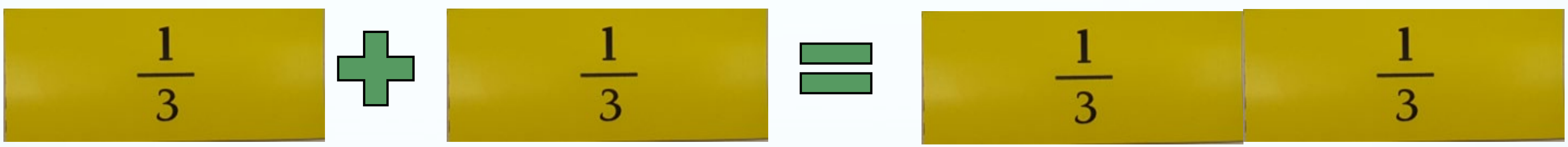


Addition with Whole Numbers



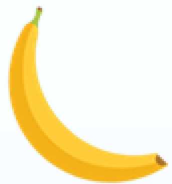
1 apple + 1 apple = 2 apples

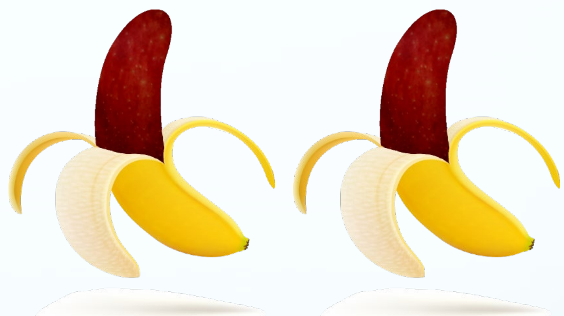
Addition with Unit Fractions and Like Denominators



1/3 + 1/3 = 2/3

Adding with Whole Numbers?



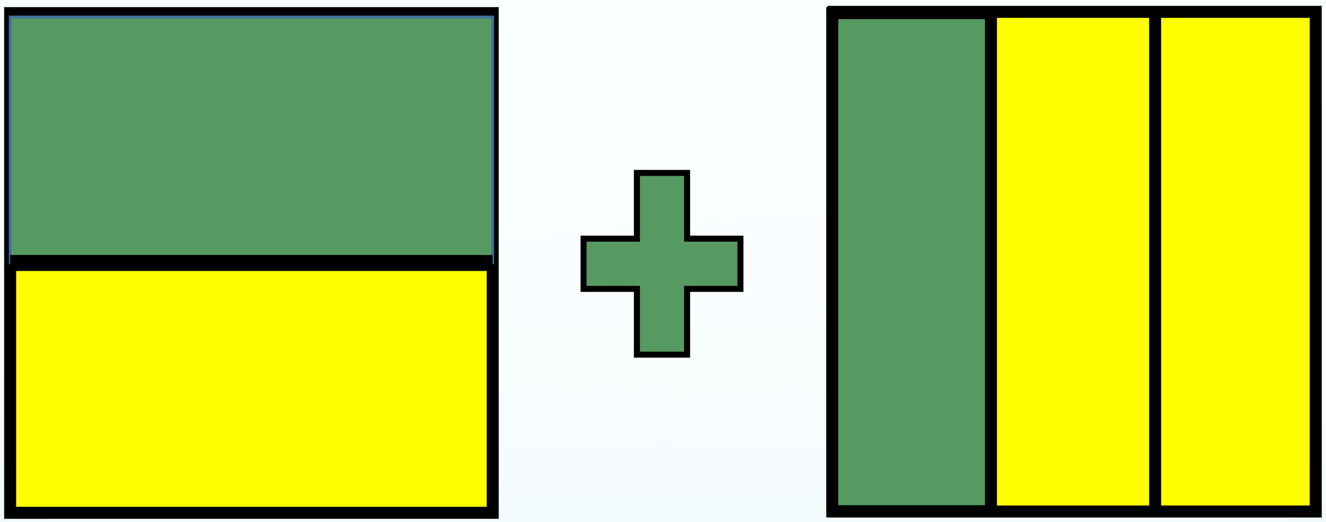


1 apple + 1 banana = 2 banapples?

Addition of Fractions: Try This

$$\frac{1}{2} + \frac{1}{3}$$

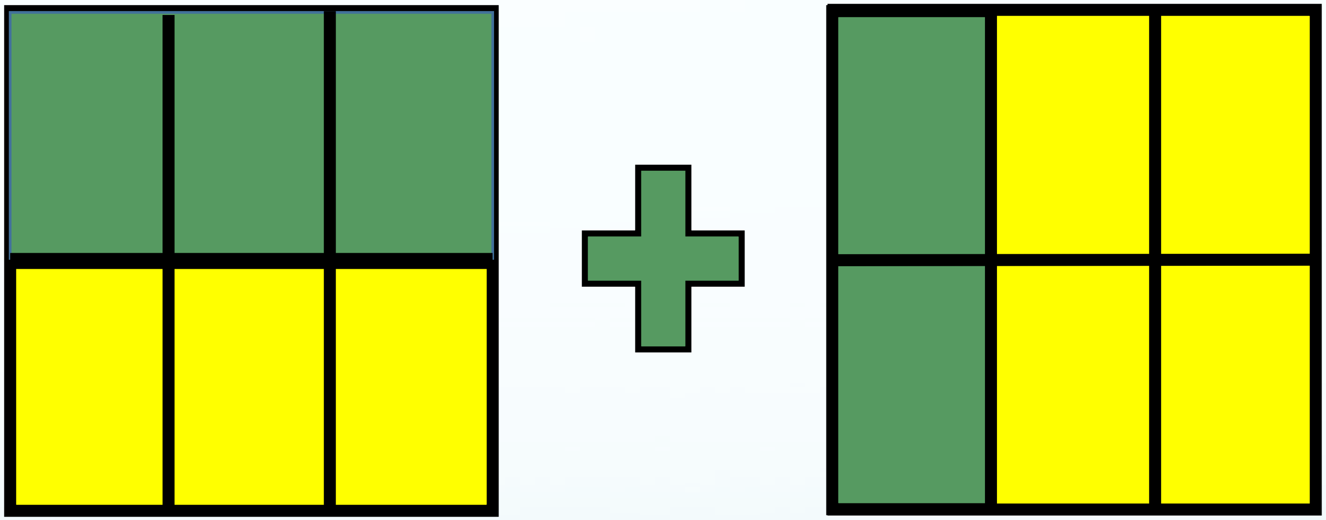
Addition with Unlike Denominators: Unit Squares?



1/2 **+** **1/3**

Adapted from Lee Stiff

Finding a Common Unit



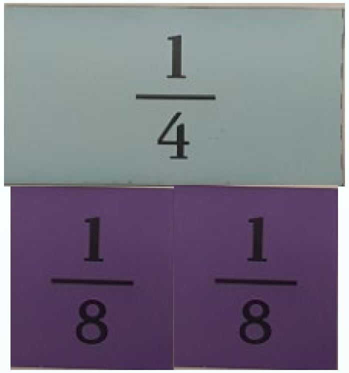
1/2 **+** **1/3**

Adapted from Lee Stiff

Equivalent Fractions

- Fractions strips are equivalent when they have the same area
  - Fractions are only equivalent when the unit whole is constant among them

Example



1/4 = 2/8

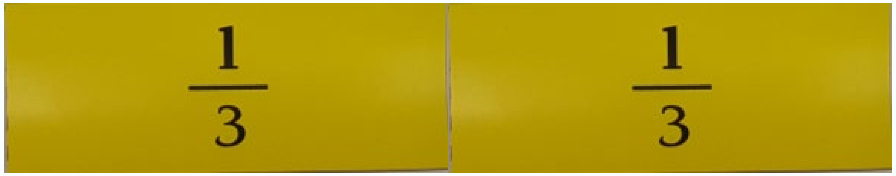
Multiplication of Whole Numbers: Equal Groups Unknown Product

2 * 3

There are 2 bags with 3 apples in each bag. How many apples are there in all?

Multiplication with a Whole Number and a Unit Fraction

2 * 1/3

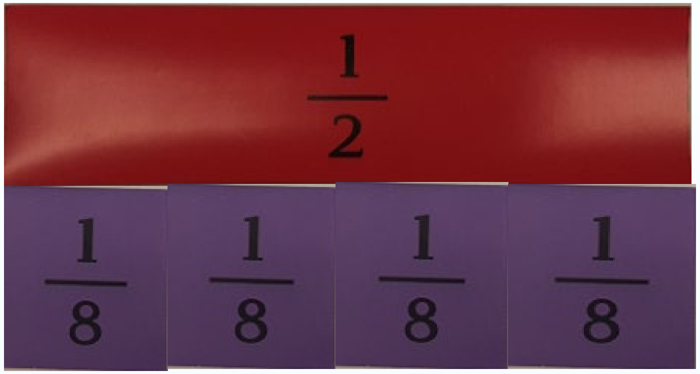


There are 2 groups of 1/3. How many 1/3 are there in all?

What does it mean to multiply a fraction by a fraction?

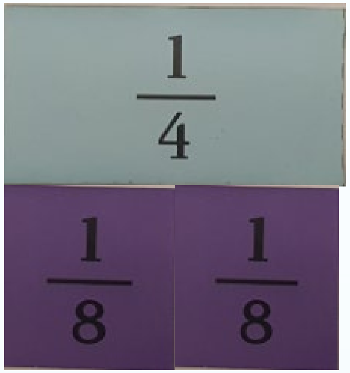
Multiplication: Both Factors a Fraction

1/4 * 1/2



Multiplication: Commutative Property

1/2 * 1/4



Back to- Division with a Fraction

- $\frac{1}{2} / \frac{1}{2} =$

- $1\frac{1}{2} / \frac{1}{2} =$

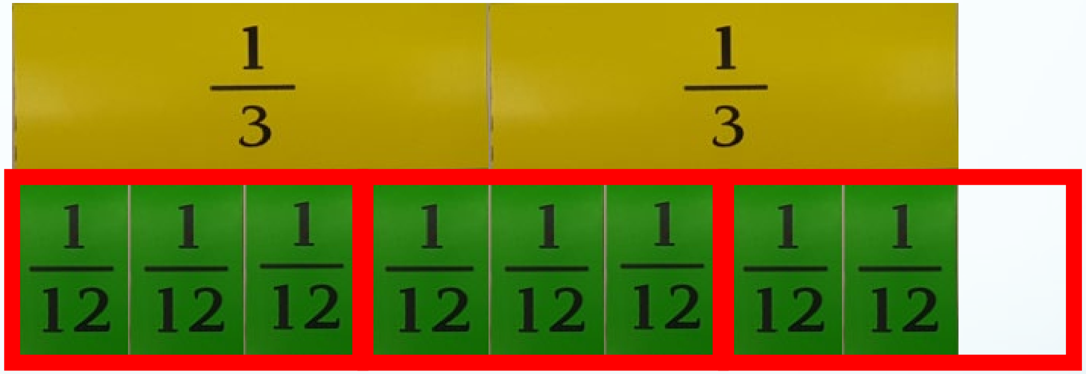
- $3 / \frac{1}{2} =$

Dividend and a Fraction Divisor

2/3 ÷ 2/12

Division with a Fraction Dividend and a Fraction Divisor

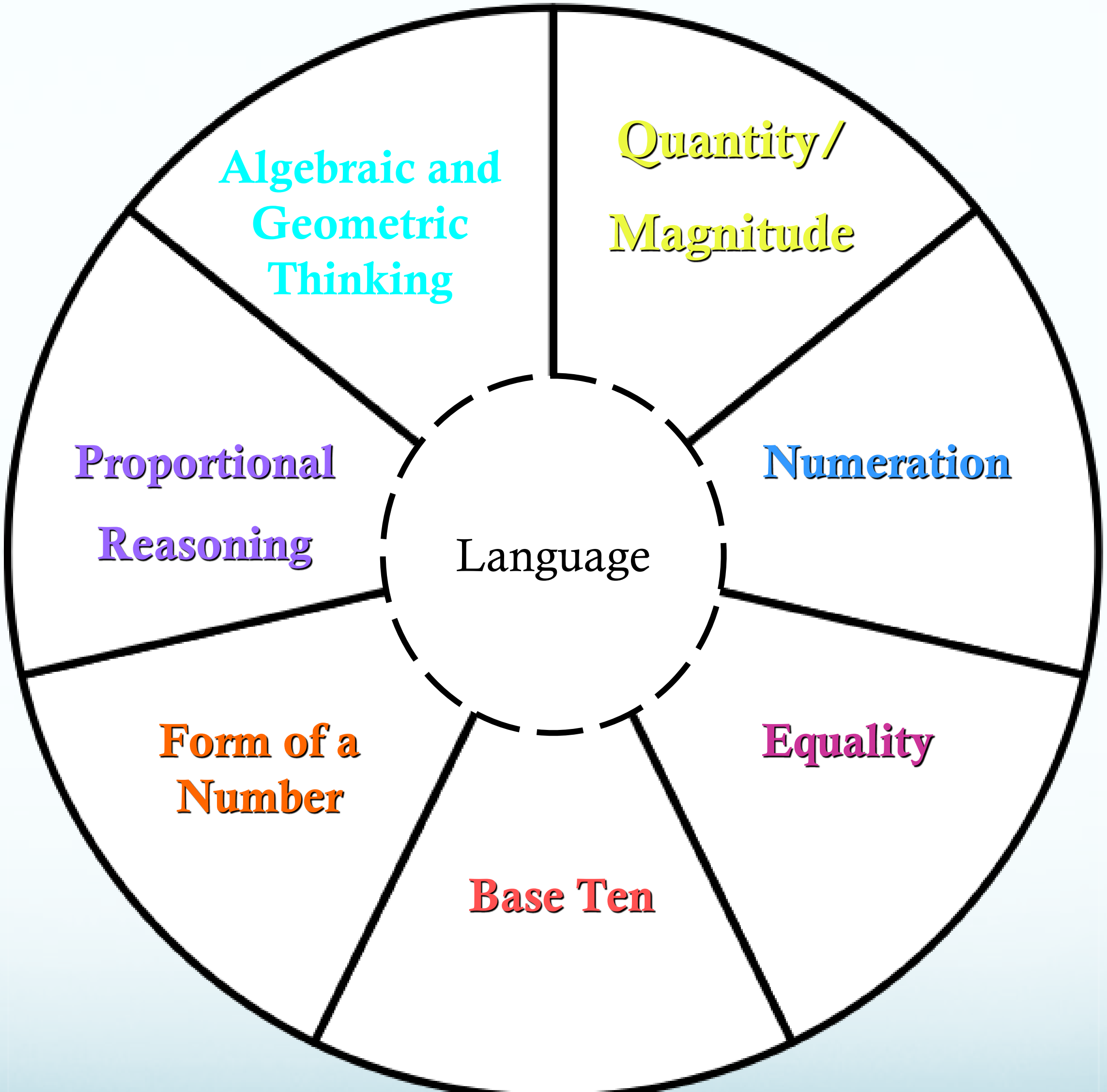
2/3 ÷ 3/12



Implementation

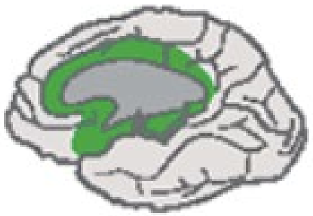
- Teacher's discussion of the Mathematical Structure is critical.

- Deborah Ball has found that teacher knowledge affects student growth.



Components of Number Sense

Universal Design for Learning Guidelines



Provide Multiple Means of

Engagement

Purposeful, motivated learners

Provide options for self-regulation
- Promote expectations and beliefs that optimize motivation
- Facilitate personal coping skills and strategies
- Develop self-assessment and reflection

Provide options for sustaining effort and persistence
- Heighten salience of goals and objectives
- Vary demands and resources to optimize challenge
- Foster collaboration and community
- Increase mastery-oriented feedback

Provide options for recruiting interest
- Optimize individual choice and autonomy
- Optimize relevance, value, and authenticity
- Minimize threats and distractions

Provide Multiple Means of

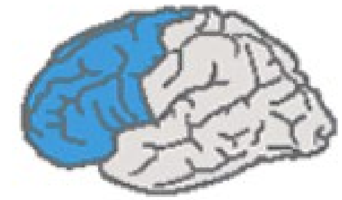
Representation

Resourceful, knowledgeable learners

Provide options for comprehension
- Activate or supply background knowledge
- Highlight patterns, critical features, big ideas, and relationships
- Guide information processing, visualization, and manipulation
- Maximize transfer and generalization

Provide options for language, mathematical expressions, and symbols
- Clarify vocabulary and symbols
- Clarify syntax and structure
- Support decoding of text, mathematical notation, and symbols
- Promote understanding across languages
- Illustrate through multiple media

Provide options for perception
- Offer ways of customizing the display of information
- Offer alternatives for auditory information
- Offer alternatives for visual information



Provide Multiple Means of

Action & Expression

Strategic, goal-directed learners

Provide options for executive functions
- Guide appropriate goal-setting
- Support planning and strategy development
- Enhance capacity for monitoring progress

Provide options for expression and communication
- Use multiple media for communication
- Use multiple tools for construction and composition
- Build fluencies with graduated levels of support for practice and performance

Provide options for physical action
- Vary the methods for response and navigation
- Optimize access to tools and assistive technologies

http://www.udlcenter.org/aboutudl/take_a_tour_udl

"

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